

# A Systematic Parameter Adaption Scheme in APSO

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**Abstract**— An adaption feature of particle swarm optimization features have better search efficiency than particle swarm optimization (PSO) is presented. It can perform a global search over the entire search space with faster convergence speed. APSO enables automatic control of weight, acceleration coefficients, and other parameters to improve efficiency and convergence speed. . Results show that APSO substantially enhances the performance of the PSO paradigm in terms of convergence speed, and solution accuracy, and algorithm reliability.

**Keywords**— Adaptive Particle Swarm Optimization; Global Optimization; Particle Swarm Optimization (PSO).

## 1. Introduction

Particle swarm optimization (PSO) was introduced by Kennedy and Eberhart in 1995 [1], [2], [3]. The PSO have a simple mechanism that mimics swarm behavior in birds flocking and fish schooling to guide the particles to search for globally optimal solutions.

Accelerating convergence speed and avoiding the local optimal have become the two most important goals in PSO research. Adaptive PSO (APSO) is formulated by developing a systematic parameter adaptation scheme and an elitist learning strategy (ELS). To enable adaptation, an evolutionary state estimation (ESE) technique is first devised. Adaptive parameter control strategies can be developed based on the identified evolutionary state and by making use of existing research results on inertia weight [13][14] and acceleration coefficients.

## 2. PSO and its Developments

**PSO Framework:** In PSO, a swarm of particles are represented as potential solutions, and each particle I is associated with two vectors, i.e., the velocity vector

$V_i = [v_{1i} v_{2i} \dots v_{Di}]$  and the position vector  $X_i = [x_{1i} x_{2i} \dots x_{Di}]$ , where D stands for the dimensions of the solution space. The velocity and the position of each particle are initialized by random vectors within the corresponding ranges. During the evolutionary process, the velocity and position of particle I on dimension d are updated as,

$$v_i^d = \omega v_i^d + c_1 r_1^d (pBest_i^d - x_i^d) + c_2 r_2^d (nBest^d - x_i^d) \quad (1)$$

$$x_i^d = x_i^d + v_i^d \quad (2)$$

Where  $\omega$  is the inertia weight [13],  $c_1$  and  $c_2$  are the acceleration coefficients [2], and  $r_1$  and  $r_2$  are two uniformly distributed random numbers independently generated within [0, 1] for the dth dimension [1]. In (1),  $pBest_i$  is the position with the best fitness found so far for theith particle, and  $nBest$  is the best position in the neighborhood. In the literature, instead of using  $nBest$ ,  $gBest$  may be used in the global-version PSO, whereas  $lBest$  may be used in the local-version PSO (LPSO).

## 3. ESE for PSO

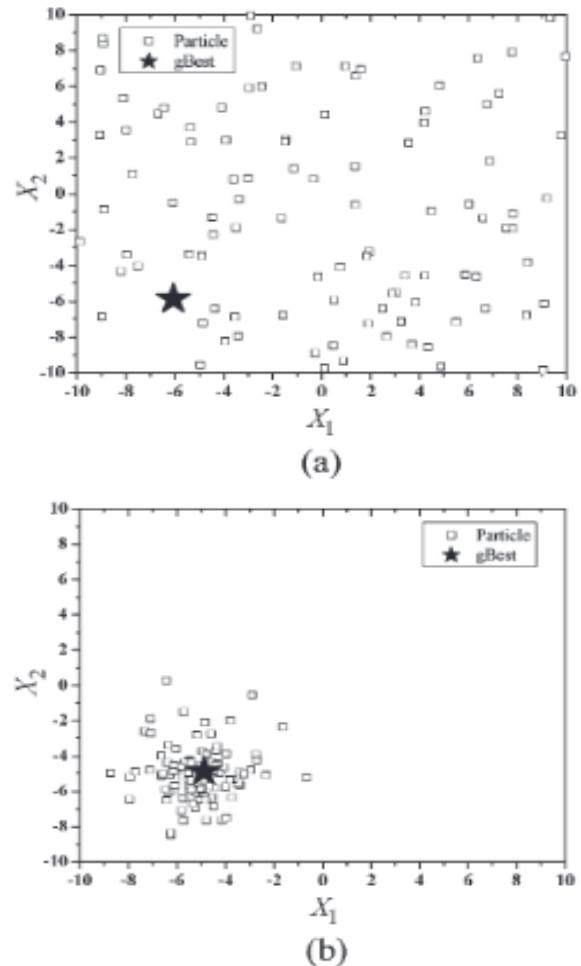


Fig.1: Population distribution observed at various stages in a PSO process. (a) Generation=1, (b) Generation=25

**ESE:** Based on the search behaviors and the population distribution characteristics of the PSO, an ESE approach is developed in this section.

The sparsely distribution information in Fig. 1 can be formulated as illustrated in Fig. 2 by calculating the mean distance of each particle to all the other particles. It is reasonable to expect that the mean instance from the globally best particle to other particles would be minimal in the convergence state since the global best tends to be surrounded by the swarm. In contrast, this mean distance would be maximal in the jumping out state, because the global best is likely to be away from the crowding swarm. Therefore, the ESE approach will take into account the population distribution information in every generation, as detailed given in the following steps.

Step 1: At the current position, calculate the mean distance of each particle  $i$  to all the other particles.

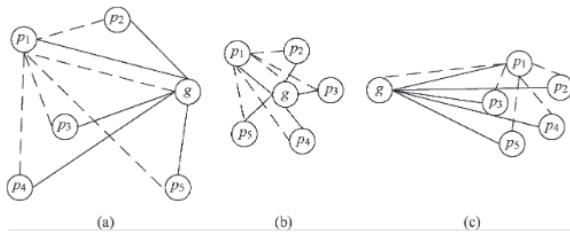


Fig.2: PSO population distribution information quantified by an evolutionary factor

For example, this mean distance can be measured using an Euclidian metric.

$$d_i = \frac{1}{N-1} \sum_{j=1, j \neq i}^N \sqrt{\sum_{k=1}^D (x_i^k - x_j^k)^2} \quad (3)$$

Where  $N$  and  $D$  are the population size and the number of dimensions, respectively.

Step 2: Denote  $d_g$  of the globally best particle as  $g$ . Compare all  $d_i$ 's, and determine the maximum and minimum distances  $d_{\max}$  and  $d_{\min}$ . Compute an “evolutionary factor”  $f$  as defined by

$$f = \frac{d_g - d_{\min}}{d_{\max} - d_{\min}} \in [0, 1]. \quad (4)$$

Take the time-varying  $f$  minimization process shown in Fig. 1 as an example to illustrate the variations of  $f$ .

Step 3: Classify  $f$  into one of the four sets S1, S2, S3, and S4, which represent the states of exploration, exploitation, convergence, and jumping out, respectively.

## 4. APSO

### 4.1 Adaptation of the Inertia Weight

The inertia weight  $\omega$  in PSO is used to balance the global and local search capabilities. Many researchers have

advocated that the value of  $\omega$  should be large in the exploration state and small in the exploitation state [4], [13], [14]. However, it is not necessarily to decrease  $\omega$  purely with time [14]. The evolutionary factor  $f$  shares some characteristics with the inertia weight  $\omega$  in that  $f$  is also relatively large during the exploration state and becomes relatively small in the convergence state. In this paper,  $\omega$  is initialized to 0.9. As  $\omega$  is not necessarily monotonic with time, but monotonic with  $f$ ,  $\omega$  will, thus, adapt to the search environment characterized by  $f$ .

In a jumping-out or exploration state [12], the large  $f$  and  $\omega$  will benefit the global search, as referenced earlier. Conversely, when  $f$  is small, an exploitation or convergence state is detected, and, hence,  $\omega$  decreases to benefit the local search.

### 4.2 Control of the Acceleration Coefficients

Adaptive control can be devised for the acceleration coefficients based on the following notion. Parameter  $c_1$  represents the “self-cognition” that pulls the particle to its own historical best position, helping explore and maintaining the diversity of the swarm. Parameter  $c_2$  represents the “social influence” that pushes the swarm to converge to the current best region globally, helping with fast convergence [4], [12]. These are two different mechanisms that should be given different treatments in different evolutionary states [10]. In this work, the acceleration coefficients are initialized to 2.0 and adaptively controlled according to the evolutionary state, with strategies developed as follows.

**Strategy 1-Increasing  $c_1$  and Decreasing  $c_2$  in an Exploration State:** It is important to explore as many optima as possible in the exploration state. Hence, increasing  $c_1$  and decreasing  $c_2$  can help particles explore individually and achieve their own historical best positions, rather than crowd around the current best particle that is likely to be associated with a local optimum.

**Strategy 2-Increasing  $c_1$  slightly and Decreasing  $c_2$  slightly in an Exploitation State:** In this state, the particles are making use of local information and grouping toward possible local optimal niches indicated by the historical best position of each particle. Hence, increasing  $c_1$  slowly and maintaining a relatively large value can emphasize the search and exploitation around  $p_B$  [9]. In the meantime, the globally best particle does not always locate the global optimal region at this stage yet. Therefore, decreasing  $c_2$  slowly and maintaining a small value can avoid the deception of a local optimum. Further, an exploitation state is more likely to occur after an exploration state and before a convergence state. Hence, changing directions for  $c_1$  and  $c_2$  should slightly be altered from the exploration state to the convergence state.

**Strategy 3-Increasing  $c_1$  slightly and Increasing  $c_2$  slightly in a Convergence State:** In the

convergence state, the swarm seems to find the globally optimal region, and, hence, the influence of  $c_2$  should be emphasized to lead other particles to the probable globally optimal region. Thus, the value of  $c_2$  should be increased. On the other hand, the value of  $c_1$  should be decreased to let the swarm converge fast. However, extensive experiments on optimizing the 12 benchmark functions given in Table I revealed that such a strategy would prematurely saturate the two parameters to their lower and upper bounds, respectively. The consequence is that the swarm will strongly be attracted by the current best region, causing premature convergence, which is harmful if the current best region is a local optimum. To avoid this, both  $c_1$  and  $c_2$  are slightly increased. Note that, slightly increasing both acceleration parameters will eventually have the same desired effect as reducing  $c_1$  and increasing  $c_2$ , because their values will be drawn to around 2.0 due to an upper bound of 4.0 for the sum of  $c_1$  and  $c_2$ .

**Strategy 4-Decreasing  $c_1$  and Increasing  $c_2$  in a Jumping-Out State:** When the globally best particle is jumping out of local optimum toward a better optimum, it is likely to be faraway from the crowding cluster. As soon as this new region is found by a particle, which becomes the (possibly new) leader, others should follow it and fly to this new region as fast as possible. A large  $c_2$  together with a relatively small  $c_1$  helps to obtain this goal. Variations of the acceleration coefficients with the evolutionary state are illustrated in Fig. 3

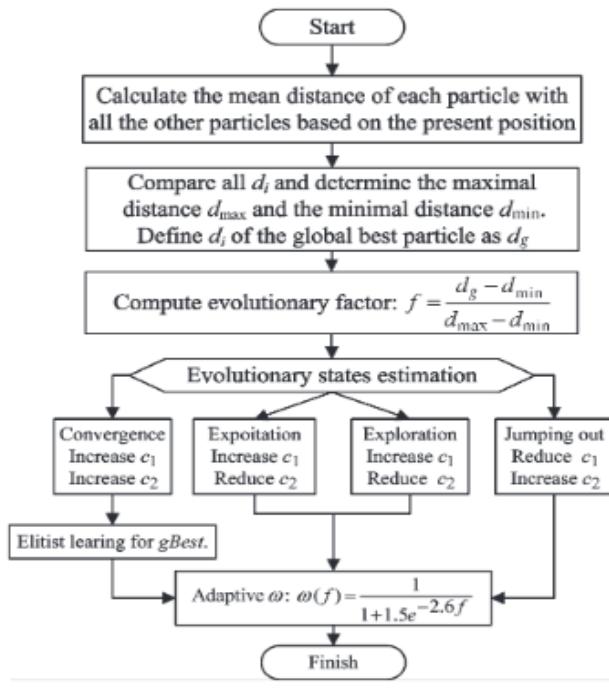


Fig. 3: ELS

The failures of using parameter adaptation alone for GPSO and VPSO on Schwefel's function suggest that a jumping-out mechanism would be necessary for enhancing

the globality of these search algorithms. Hence, an "ELS" is designed here and applied to the globally best particle so as to help jump out of local optimal regions when the search is identified to be in a convergence state [5]. Unlike the other particles, the global leader has no exemplars to follow. It needs fresh momentum to improve itself. Hence, a perturbation-based ELS is developed to help *gBest* push itself out to a potentially better region. If another better region is found, then the rest of the swarm will follow the leader to jump out and converge to the new region. The ELS randomly chooses one dimension of *gBest*'s historical best position, which is denoted by  $P^d$  for the  $d$ th dimension. Only one dimension is chosen because the local optimum is likely to have some good structure of the global optimum, and, hence, this should be protected. As every dimension has the same probability to be chosen, the ELS operation can be regarded to perform on every dimension in a statistical sense. Similar to simulated annealing, the mutation operation in evolutionary programming or in evolution strategies, elitist learning is performed through a Gaussian perturbation.

$$P^d = P^d + (X_{\max}^d - X_{\min}^d) \cdot \text{Gaussian}(\mu, \sigma^2). \quad (5)$$

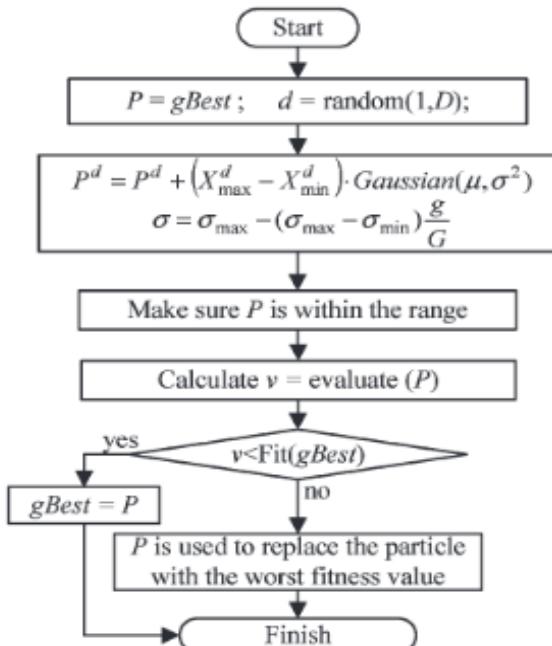


Fig.4: Search Behaviors of APSO

The search range [ $X_{\min}$ ,  $X_{\max}$ ] is the same as the lower and upper bounds of the problem. The  $\text{Gaussian}(\mu, \sigma^2)$  is a random number of a Gaussian distribution with a zero mean and a standard deviation (SD)  $\sigma$ , which is termed as the "Elitist learning rate." Similar to some time-varying neural network training schemes, it is suggested that  $\sigma$  be decreased with the generation number, which is given by [8]

$$\sigma = \sigma_{\max} - (\sigma_{\max} - \sigma_{\min}) \frac{g}{G} \quad (6)$$

Where  $\sigma_{\max}$  and  $\sigma_{\min}$  are the upper and lower bounds of  $\sigma$ , which represents the learning scale to reach a new region. Empirical study shows that  $\sigma_{\max}=1.0$  and  $\sigma_{\min}=0.1$  result in good performance on most of the test functions (refer to Section VI-C for an in-depth discussion). Alternatively,  $\sigma$  may geometrically be decreased, similar to the temperature-decreasing scheme in Boltzmann learning seen in simulated annealing. The ELS process is illustrated in Fig.4.

The complete flowchart of the APSO algorithm with adaptive parameters and ELS is shown in Fig.5.

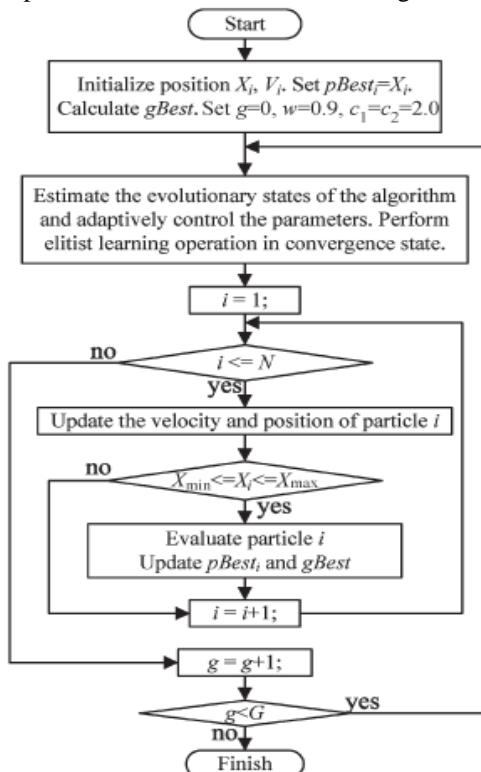


Fig.5: Adaptive parameters and ELS

Before applying the APSO to comprehensive tests on benchmark functions, we first investigate its search behaviors in unimodal and multimodal search spaces.

*APSO in Unimodal Search Space:* The search behavior of the APSO in a unimodal space has been investigated on the Sphere function (*f1*)[6]. In a unimodal space, it is important for an optimization or search algorithm to converge fast and to refine the solution for high accuracy. The inertia weight confirms that the APSO maintains a large  $\omega$  in the exploration phase (for about 50 generations), and then a rapidly decreasing  $\omega$  follows exploitation, leading to convergence, as the unique global optimum region is found by a leading particle, and the swarm follows it. The ESE in APSO has influenced

the acceleration coefficients. The curves for  $c1$  and  $c2$  somewhat show good agreement with the ones given in Fig. 5. It can be seen that  $c1$  increases while  $c2$  decreases in the exploration and exploitation phases. Then,  $c1$  and  $c2$  reverse their directions when the swarm converges, eventually returning to around 2.0. Then, trials in elitist learning perturb the particle that leads the swarm, which is reflected in the slight divergence between  $c1$  and  $c2$  that follows. The search behavior on the unimodal function indicates that the proposed APSO algorithm has indeed identified the evolutionary states and can adaptively control the parameters for improved performance.

*APSO in Multimodal Search Space:* Here, the APSO is tested again to see how it will adapt itself to a multimodal space. When solving multimodal functions, a search algorithm should maintain diversity of the population and search for as many optimal regions as possible. The search behavior of the APSO is investigated on Rastrigin's function (*f8*)[4],[7]. To compare the diversity by the APSO and the traditional PSO, a yardstick proposed in [11] is used here, called the "population standard deviation," (psd)

$$psd = \sqrt{\left[ \sum_{i=1}^N \sum_{j=1}^D (x_i^j - \bar{x}^j)^2 \right] / (N-1)} \quad (7)$$

Where  $N$ ,  $D$ , and  $x$  are the population size, the number of dimension, and the mean position of all the particles, respectively.

The variations in  $psd$  can indicate the diversity level of the swarm. If  $psd$  is small, then it indicates that the population has closely converged to a particular region, and the diversity of the population is low. A larger value of  $psd$  indicates that the population is of higher diversity. However, it does not mean that a larger  $psd$  is always better than a smaller one because an algorithm that cannot converge may also present a large  $psd$ . Hence, the  $psd$  needs to be considered together with the solution that the algorithm arrives at. The results of  $psd$  comparisons are plotted and those of the evolutionary processes. It can be seen that the APSO has an ability to get out from the local optima, which is reflected by the regained diversity of the population, as revealed in with a steady improvement in the solution, the inertia weight and the acceleration coefficient behaviors of the APSO, respectively. These plots confirm that, in a multimodal space, the APSO can also find a potential optimal region (maybe a local optimum) fast in an early phase and converge fast with a rapid decreasing diversity due to the adaptive parameter strategies. However, if the current optimal region is local, then the swarm can separate and jump out. Hence, the APSO can appropriately increase the diversity of the population so as to explore for a better region owing to the ELS in the convergence state. This behavior with adaptive population diversity is valuable for a global search algorithm is used to prevent from being trapped in the local optima and to find the global optimum in a multimodal space.

## 5. Analysis of Parameter Adaptationand Elitist Learning

### 5.1 Merits of Parameter Adaptation and Elitist Learning

To quantify the significance of these two operations, the performance of APSO without parameter adaptation or elitist learning is tested under the same running conditions. Results of the mean values on 30 independent trials are presented in [2], [10]. It is clear from the results that with elitist learning alone and without adaptive control of parameters, the APSO can still deliver good solutions to multimodal functions. However, the APSO suffers from lower accuracy in solutions to unimodal functions. As algorithms can easily locate the global optimal region of a unimodal function and then refine the solution, the lower accuracy may be caused by the slower convergence speed to reach the global optimal region. On the other hand, the APSO with parameter adaptation alone but without ELS can hardly jump out of the local optima and, hence, results in poor performance on multimodal functions. However, it can still solve unimodal problems well. Note that both of the reduced APSO algorithms generally outperform a standard PSO that involves neither adaptation parameters nor elitist learning. However, the full APSO is the most powerful and robust for any tested problem. This is most evident in the test results on f4. These results together with the results confirm the hypothesis that parameter adaptation speeds up the convergence of the algorithm and elitist learning helps the swarm jump out of the local optima and find better solutions.

### 5.2 Sensitivity of the Acceleration Rate

The effect of the acceleration rate, which is reflected by its bound  $\delta$ , on the performance of the APSO is investigated here. For this, the learning rate  $\sigma$  is, hence, fixed (e.g.,  $\sigma_{\max} = \sigma_{\min} = 0.5$ ), and the other parameters of the APSO remain the same as in Section V-A. The investigation consists of six test strategies for  $\delta$ , the first three being to fix its value to 0.01, 0.05, and 0.1, respectively, and the remaining three being randomly to generate its value using a uniform distribution within [0.01, 0.05], [0.05, 0.1], and [0.01, 0.1], respectively. The results are presented in Table X in terms of the mean values of the solutions found in 30 independent trials. It can be seen that APSO is not very sensitive to the acceleration rate  $\delta$ , and the six acceleration rates all offer good performance. This may be owing to the use of bounds for the acceleration coefficients and the saturation to restrict their sum by (12). Therefore, given the bounded values of  $c_1$  and  $c_2$  and their sum restricted by (12), an arbitrary value within the range [0.05, 0.1] for  $\delta$  should be acceptable to the APSO algorithm.

## 6. Conclusion

In this paper, PSO has been extended to APSO. This progress in PSO has been made possible by ESE, which utilizes the population distribution information and fitness of relative particles, sharing a similar spirit to the internal modelling in evolution strategies. ESE-based parameter adaptation technique departs from the existing parameter variation schemes, based on the generation number alone. Hence, the APSO is still simple and almost as easy to use as the standard PSO, whereas it brings in substantially improved performance in terms of convergence speed, global optimality, solution accuracy, and algorithm reliability.

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