Special Issue of International Journal of Linguistics and Computational Applications ISSN 2394-6393 (Online) ISSN 2394-6385 (Print)
Proceedings of the UGC Sponsored - National Conference on "Recent Statistical Computing Techniques and their Applications"
Department of Statistics and Department of Computer Science, Ramanujan College,University of Delhi, Delhi.
March 11-12, 2016. **Published on January 2017**

Shrinkage Estimators of the Reliability Characteristics of Generalized Half Logistic Distribution

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Abstract— Measure of reliability P = P(X>Y) is considered. Shrinkage estimators are considered for the powers of parameter, 'P' under type I and type II censorings. Simulation study is conducted to judge the performance of estimators.

Keywords— Generalized half logistic distribution (GHLD); shrinkage estimation; type I and type II censorings; p-value.

1. Introduction

The reliability function R(t) is defined as the probability of failure-free operation until time t. Thus, if the random variable (rv) X denotes the lifetime of an item or system, then R(t) = P(X>t). Another measure of reliability under stress-strength set-up is the probability P = P(X>Y), which represents the reliability of an item or system of random strength X subject to random stress Y. A lot of work has been done in the literature for the estimation and testing of parameter, R(t) and 'P' under censorings and complete sample case for individual distributions. For a brief review, one may refer to Pugh (1963), Basu (1964), Bartholomew (1957, 1963), Tong (1974, 1975), Johnson (1975), Kelly, Kelly and Schucany (1976), Sathe and Shah (1981), Chao (1982), Constantine, Karson and Tse (1986), Awad and Gharraf (1986), Tyagi and Bhattacharya (1989), Chaturvedi and Rani (1997,1998), Chaturvedi and Surinder (1999), Chaturvedi and Tomer (2002, 2003), Chaturvedi and Singh (2006, 2008), Chaturvedi and Pathak (2012, 2013, 2014), and others.

Thompson (1968) introduced the concept of 'shrinkage estimators'. A lot of work has been done in the literature in the direction of shrinkage estimators. For some citations, one may refer to George (1986), Ebrahimi and Hosmane (1987) , Ghosh, Nickerson and Sen (1987), Blattberg & George (1991), Clyde, Parmigiani and Vidakovic (1998), Kubokawa (1998), Kolaczyk(1999), Longford (1999), Ahmed (2001), Royle and Link (2002), Sendur and Selesnick (2002), Fourdrinier, Strawderman, and Wells (2003), Pope and Szapudi (2008), Prakash and Singh (2008), Chen , Wiesel and O. Hero (2009, 2010), Ledoit and Wolf (2012), Carreras and Brannath (2013), Liao (2013), Cheng and Liao (2014, 2016), Lu and Su (2015). Pandey (1983) proposed various shrinkage estimators for the mean of exponential distribution. Tse and Tso (1996), Baklizi (2003) and Baklizi and Abu Dayyeh (2003) proposed shrinkage estimators of R(t) and 'P' for one-parameter exponential distribution. For estimating R(t), type I and type II censorings were considered. In order to estimate 'P', complete sample case was considered.

Half logistic model, obtained as the distribution of the absolute standard logistic variate, is probability model considered by Balakrishnan (1985). Balakrishnan and Hossain (2007) considered generalized (Type II) version of logistic distribution and derived some interesting properties of the distribution. Ramakrishna (2008) considered two generalized versions of HLD namely Type I and Type II along with point estimation of scale parameters and estimation of stress strength reliability based on complete sample.

Let the life X of an item have the GHLD, then cumulative distribution function (cdf) and probability density function (pdf) of the random variable (rv) X are, respectively

$$F(x;\ \lambda) = 1 - \left(\frac{2\,e^{-x}}{1+e^{-x}}\right)^{\lambda} \quad ;\ x > 0,\ \lambda > 0\ \text{, and}$$

$$f(x;\ \lambda) = \frac{\lambda}{1+e^{-x}}\left(\frac{2e^{-x}}{1+e^{-x}}\right)^{\lambda} \quad ;\ x > 0,\ \lambda > 0\,. \qquad (1.1)$$

Here, it should be noted that λ is the shape parameter and, for λ=1, it comes out to be the half-logistic distribution. Let the rv X follow $f(x;\ a_1, \lambda_1, \underline{\theta}_1)$ distribution and Y follow $f(y;\ a_2, \lambda_2, \underline{\theta}_2)$ distribution. Then, we have

$$P = \frac{\lambda_1}{\lambda_1 + \lambda_2}\,. \qquad (1.2)$$

Now, we summarize the results of Chaturvedi, Kang and Pathak (2016). Suppose, n items are put on a test and the test is terminated after the first r ordered observations are recorded. Let $0 \le X_{(1)} \le X_{(2)} \le \ldots \le X_{(r)},\ 0 < r < n$, be the lifetimes of first r ordered observations. Obviously, (n-r) items survived until $X_{(r)}$. Let

$$S_r = \sum_{i=1}^{r} \ln\left\{\frac{1}{2}\left(e^{x_{(i)}}+1\right)\right\} + (n-r)\ln\left\{\frac{1}{2}\left(e^{x_{(r)}}+1\right)\right\} .$$

The likelihood function is

$$L(\lambda \mid x_{(1)}, x_{(2)}, ..., x_{(r)}) = n(n-1).....(n-r+1)\,\lambda^r \prod_{i=1}^{r}\left(\frac{1}{1+e^{-x_{(i)}}}\right)\exp(-\lambda s_r).$$

(1.3)

S_r is complete and sufficient for the family of distribution given at (1.1) and the pdf of S_r is

$$g(s_r;\lambda) = \frac{\lambda^r s_r^{r-1}\exp(-\lambda s_r)}{\Gamma(r)};\ r>0,\ \lambda>0,\ 0<s_r<\infty .$$

(1.4)

The maximum likelihood estimators (MLES) of λ and 'P' are, respectively,

$$\hat{\lambda}_{II} = \frac{r}{S_r}, \quad (1.5)$$

and

$$\hat{P}_{II} = \frac{\hat{\lambda}_{1II}}{\hat{\lambda}_{1II}+\hat{\lambda}_{2II}}, \quad (1.6)$$

where, for

$$S_{r_1} = \sum_{i=1}^{r_1} \ln\left\{\frac{1}{2}\left(e^{x_{(i)}}+1\right)\right\} + (n-r_1)\ln\left\{\frac{1}{2}\left(e^{x_{(r1)}}+1\right)\right\}$$ and

$$T_{r_2} = \sum_{j=1}^{r_2} \ln\left\{\frac{1}{2}\left(e^{x_{(j)}}+1\right)\right\} +$$

$$(m-r_2)\ln\left\{\frac{1}{2}\left(e^{x_{(r_2)}}+1\right)\right\},\ \hat{\lambda}_{1II} = \frac{r_1}{S_{r_1}}$$ and $$\hat{\lambda}_{2II} = \frac{r_2}{T_{r_2}}$$ .For

$q \in (-\infty, \infty)$, $q \neq 0$, the uniformly minimum variance unbiased estimator (UMVUE) of λ^q is

$$\tilde{\lambda}^q_{II} = \begin{cases} \dfrac{\Gamma(r)}{\Gamma(r-q)} S_r^{-q} & (q<r) \\ 0, \text{otherwise.} \end{cases} \quad (1.7)$$

The UMVUE of 'P' is given by

$$\tilde{P}_{II} = \begin{cases} (r_2-1)\sum_{i=0}^{r_2-2}(-1)^i\binom{r_2-2}{i}\left(\dfrac{S_{r_1}}{T_{r_2}}\right)^{i+1} B(i+1, r_1),\ S_{r_1}<T_{r_2} \\ (r_2-1)\sum_{i=0}^{r_1-1}(-1)^i\binom{r_1-1}{i}\left(\dfrac{T_{r_2}}{S_{r_1}}\right)^{i} B(i+1, r_2-1),\ T_{r_2}<S_{r_1}. \end{cases}$$

(1.8)

Now, we consider the case of type I censoring. Let $0 \le X_{(1)} \le X_{(2)} \le ... \le X_{(n)}$ be the failure times of n items under test from (1.1). The test begins at time $X_{(0)} = 0$ and the system operates till $X_{(1)} = x_{(1)}$ when the first failure occurs. The failed item is replaced by a new one and the system operates till the second failure occurs at time $X_{(2)} = x_{(2)}$, and so on. The experiment is terminated at time t_o. If $N(t_o)$ be the number of failures during the interval $[0, t_o]$, then

$$P[N(t_o) = r \mid t_o] = \frac{\left(n\lambda \ln\left\{\frac{1}{2}\left(e^{t_o}+1\right)\right\}\right)^r}{r!}\exp\left(-n\lambda\ln\left\{\frac{1}{2}\left(e^{t_o}+1\right)\right\}\right).$$

(1.9)

The MLES of λ and 'P' are, respectively,

$$\hat{\lambda}_I = r\left(n\ln\left\{\frac{1}{2}\left(e^{t_o}+1\right)\right\}\right)^{-1},$$

(1.10)

and $$\hat{P}_I = \frac{\hat{\lambda}_{1I}}{\hat{\lambda}_{1I}+\hat{\lambda}_{2I}}, \quad (1.11)$$

where $$\hat{\lambda}_{1I} = \frac{r_1}{n\ln\left\{\frac{1}{2}\left(e^{t_o}+1\right)\right\}}$$ and

$$\hat{\lambda}_{2I} = \frac{r_2}{m\ln\left\{\frac{1}{2}\left(e^{t_o}+1\right)\right\}}.$$

The UMVUES of λ^q and 'P' are, respectively,

$$\tilde{\lambda}^q_I = \begin{cases} \dfrac{r!}{(r-q)!}\left\{n\ln\left\{\frac{1}{2}\left(e^{t_o}+1\right)\right\}\right\}^{-q} & (q \le r) \\ 0, \text{otherwise} \end{cases}, \quad (1.12)$$

and

$$\tilde{P}_I = \begin{cases} r_2\sum_{i=0}^{r_1}(-1)^i\binom{r_1}{i}\left(\dfrac{m}{n}\right)^i B(i+1, r_2), & m<n \\ r_2\sum_{i=0}^{r_2-1}(-1)^i\binom{r_2-1}{i}\left(\dfrac{n}{m}\right)^{i+1} B(i+1, r_1+1), & n<m. \end{cases}$$

(1.13)

In Section 2, we propose shrinkage estimators for the powers of λ. We consider estimation of powers of λ because they come in expressions for the moments of different distributions and hazard-rate. In Section 3, we develop shrinkage estimators of 'P'. Finally, in Section 4, numerical findings are presented.

2. Shrinkage Estimators of Powers of λ

We first consider the shrinkage estimator of λ^q based on its MLE and type II censored data. Let λ_o be the guess value of λ. We consider,

$$\hat{\hat{\lambda}}^q_{II} = \alpha_1\hat{\lambda}^q_{II} + (1-\alpha_1)\lambda_o^q,\ 0 \le \alpha_1 \le 1. \quad (2.1)$$

The value of α_1 that minimizes the mean sum of squares due to error (MSE) of $\hat{\hat{\lambda}}^q_{II}$ is

$$\alpha_1 = \frac{(\lambda^q - \lambda^q_o)\,[\,E(\hat{\lambda}^q_{II}) - \lambda^q_o\,]}{E(\hat{\lambda}^{2q}_{II}) + \lambda^{2q}_o - 2\,\lambda^q_o\,E(\hat{\lambda}^q_{II})}. \qquad (2.2)$$

Using (1.4) and (1.5) ,

$$E(\hat{\lambda}^q_{II}) = \left\{\frac{r^q\,\Gamma(r - q)}{\Gamma(r)}\right\}\lambda^q \qquad (r > q)$$

and $$E(\hat{\lambda}^{2q}_{II}) = \left\{\frac{r^{2q}\,\Gamma(r - 2q)}{\Gamma(r)}\right\}\lambda^{2q} \qquad (r > 2q).$$

Since, λ is unknown, we estimate it by $\hat{\lambda}_{II}$. Now, we propose shrinkage estimator using the p-value of the likelihood ratio test. Consider $H_o : \lambda = \lambda_o$ against $H_1 : \lambda \neq \lambda_o$. From (1.3), H_o is rejected when $s_r < \dfrac{\chi^2_{(2r)}\,(1 - \frac{\alpha}{2})}{2\lambda_o}$ or $s_r > \dfrac{\chi^2_{(2r)}\,(\frac{\alpha}{2})}{2\lambda_o}$. Let τ_1 be the observed value of $2\lambda_o S_r$. Then, the p-value for this test is $z_1 = 2\min\{1 - F(\tau_1), F(\tau_1)\}$, where $F(\tau_1)$ is the cumulative distribution function of $2\lambda_o S_r$. Since a large value of z_1 indicates that λ is close to its guess value λ_o [see Tse and Tso (1996)], we can use z_1 to form the shrinkage estimator

$$\hat{\hat{\lambda}}^q_{II(z_1)} = (1 - z_1)\,\hat{\lambda}^q_{II} + z_1\,\lambda^q_o. \qquad (2.3)$$

Now, we consider shrinkage estimator of λ^q based on its UMVUE and type II censored data. We propose

$$\tilde{\tilde{\lambda}}^q_{II} = \alpha_2\,\tilde{\lambda}^q_{II} + (1 - \alpha_2)\,\lambda^q_o\,,\ 0 \le \alpha_2 \le 1. \qquad (2.4)$$

The value of α_2 which minimizes variance of $\tilde{\tilde{\lambda}}^q_{II}$ is

$$\alpha_2 == \frac{(\lambda^q - \lambda^q_o)^2}{\left\{\frac{\Gamma(r)\ \Gamma(r - 2q)}{\Gamma^2(r - q)}\right\}\lambda^{2q} - 2\,\lambda^q_o\,\lambda^q + \lambda^{2q}_o}. \qquad (2.5)$$

Since powers of λ are unknown, we replace them by their UMVUES given at (1.7). Based on p-value z_1 already defined,

$$\tilde{\tilde{\lambda}}^q_{II(z_1)} = (1 - z_1)\,\tilde{\lambda}^q_{II} + z_1\,\lambda^q_o. \qquad (2.6)$$

Let us define the shrinkage estimator of λ^q based on its MLE and type 1 censored data to be

$$\hat{\hat{\lambda}}^q_I = \alpha_3\,\hat{\lambda}^q_I + (1 - \alpha_3)\,\lambda^q_o\,,\ 0 \le \alpha_3 \le 1. \qquad (2.7)$$

The value of α_3 which minimizes the MSE of is given by $\hat{\hat{\lambda}}^q_I$

$$\alpha_3 = \frac{(\lambda^q - \lambda^q_o)\,[E(\hat{\lambda}^q_I) - \lambda^q_o]}{E(\hat{\lambda}^{2q}_I) + \lambda^{2q}_o - 2\,\lambda^q_o\,E(\hat{\lambda}^q_I)}. \qquad (2.8)$$

Since, λ is unknown, we use its MLE.

The critical region of the likelihood ratio test for testing $H_0 : \lambda = \lambda_o$ against $H_1 : \lambda \neq \lambda_o$ under type I censoring is given by $\{r \le a \text{ or } r \ge b\}$, where 'a' and 'b' are chosen such that

$$P_{Ho}\,(r \le a) + P_{Ho}\,(r \ge b) = \alpha\,.$$

If z_2 be the p-value of the test, then the shrinkage estimator of λ^q is

$$\hat{\hat{\lambda}}^q_{I(z_2)} = (1 - z_2)\,\hat{\lambda}^q_I + z_2\lambda^q_o. \qquad (2.9)$$

The shrinkage estimator of λ^q based on its UMVUE and type I censored data is

$$\tilde{\tilde{\lambda}}^q_I = \alpha_4\,\tilde{\lambda}^q_I + (1 - \alpha_4)\,\lambda^q_o\,,\ 0 \le \alpha_4 \le 1, \qquad (2.10)$$

where

$$\alpha_4 = \frac{(\lambda^q - \lambda^q_o)[E(\tilde{\lambda}^q_I) - \lambda^q_o]}{E(\tilde{\lambda}^q_I)^2 + \lambda^{2q}_o - 2\lambda^q_o E(\tilde{\lambda}^q_I)}. \qquad (2.11)$$

Here,

$$E(\tilde{\lambda}^q_I)^2 = \frac{(q!)^2}{\left\{n\ \ln\left\{\frac{1}{2}\left(e^{t_o} + 1\right)\right\}\right\}^{2q}}\exp\left\{-\,n\,\lambda\,\ln\left\{\frac{1}{2}\left(e^{t_o} + 1\right)\right\}\right\}\sum_{r=q}^{\infty}\binom{r}{q}^2\frac{\left(n\,\lambda\,\ln\left\{\frac{1}{2}\left(e^{t_o} + 1\right)\right\}\right)^r}{r!}.$$

Based on p-value z_2 already defined,

$$\tilde{\tilde{\lambda}}^q_{I(z_2)} = (1 - z_2)\,\tilde{\lambda}^q_I + z_2\lambda^q_o. \qquad (2.12)$$

3. Shrinkage Estimators of 'P'

For $\hat{P}_{II}$ defined in (1.6), we propose the shrinkage estimator of 'P' to be

$$\hat{\hat{P}}_{II} = \alpha_9\,\hat{P}_{II} + (1 - \alpha_9)\,P_o\,,\ 0 \le \alpha_9 \le 1. \qquad (3.1)$$

The value of α_9 , which minimizes the MSE of $\hat{\hat{P}}_{II}$ is given by

$$\alpha_9 = \frac{[P - P_o]\,[E(\hat{P}_{II}) - P_o]}{[E(\hat{P}_{II})^2 - 2P_o\,E(\hat{P}_{II}) + P_o^2]}. \qquad (3.2)$$

We can write (1.6) as

$$\hat{P}_{II} = \left(1 + \frac{\hat{\lambda}_{2II}}{\hat{\lambda}_{1II}}\right)^{-1}$$

$$= \left[1 + \frac{\lambda_2 r_1}{\lambda_1 r_2} F(2r_1, 2r_2)\right]^{-1} ,$$

where the rv $F(2r_1, 2r_2)$ follows F-distribution with $(2r_1, 2r_2)$ degrees of freedom and having the pdf

$$f(F) = \frac{\left(\frac{r_1}{r_2}\right)^{r_1}}{B(r_1, r_2)} \cdot \frac{F^{r_1 - 1}}{\left[1 + \frac{r_1}{r_2} F\right]^{r_1 + r_2}} ; 0 < F < \infty .$$

Making the transformation

$$\left(1 + \frac{\lambda_2}{\lambda_1} F\right)^{-1} = Q ,$$

the pdf of Q comes out to be

$$f(q) = \frac{\left(\frac{r_2 \lambda_1}{r_1 \lambda_2}\right)^{r_2}}{B(r_1, r_2)} \cdot \frac{q^{r_2 - 1} (1 - q)^{r_1 - 1}}{\left[1 + \left(\frac{r_2 \lambda_2}{r_1 \lambda_1} - 1\right) q\right]^{r_1 + r_2}} ; 0 < q < 1 . \tag{3.3}$$

The distributions for which $r_1\lambda_1 = r_2\lambda_2$,

$$f(q) = \frac{1}{B(r_1, r_2)} q^{r_2 - 1} (1 - q)^{r_1 - 1} ; 0 < q < 1$$

and

$$E(Q^l) = \frac{B(r_1, r_2 + l)}{B(r_1, r_2)} .$$

If $r_1\lambda_1 \neq r_2\lambda_2$, then from (3.3) ,

$$E(Q^l) = \frac{\left(\frac{r_2 \lambda_1}{r_1 \lambda_2}\right)^{r_2}}{B(r_1, r_2)} \int_0^1 \frac{q^{l + r_2 - 1} (1 - q)^{r_1 - 1}}{\left[1 + \left(\frac{r_2 \lambda_2}{r_1 \lambda_1} - 1\right) q\right]^{r_1 + r_2}} .$$

Putting

$$1 + \left(\frac{r_2 \lambda_2}{r_1 \lambda_1} - 1\right) q = u ,$$

$$E(Q^l) = \frac{1}{B(r_1, r_2)} (-1)^{l + r_2 - 1} \left(\frac{r_1 \lambda_1}{r_2 \lambda_2}\right)^l \left(1 - \frac{r_1 \lambda_1}{r_2 \lambda_2}\right)^{-r_1 - r_2 - l + 1} \sum_{j=0}^{r_1 - 1} (-1)^j \binom{r_1 - 1}{j} \left(\frac{r_1 \lambda_1}{r_2 \lambda_2}\right)^j$$

$$\sum_{k=0}^{l + r_2 - 1} (-1)^k \binom{l + r_2 - 1}{k} I\left(\frac{r_2 \lambda_2}{r_1 \lambda_1}; j + k - r_1 - r_2\right) (c \neq 1) .$$

Here,

$$I(c, p) = \int_1^c t^p \, dt$$

$$= \begin{cases} (c^{p+1} - 1)/(p + 1) ; & p \neq 1 \\ \log c ; & p = -1 . \end{cases}$$

Since, λ_1 and λ_2 are unknown, they are estimated by their MLES.

Now, we propose shrinkage estimator of 'P' based on the p-value related to the likelihood ratio test of the hypothesis $H_o : P = P_o$ against the alternative $H_1 : P \neq P_o$. For $k = P_o / (1 - P_o)$, these hypotheses are equivalent to $H_0 : \lambda_1 = k \lambda_2$ against the alternative $H_1 : \lambda_1 \neq k \lambda_2$. Denoting by Θ_o and Θ , respectively, the parametric space restricted by the null hypothesis and the entire parametric space, it can be seen that the likelihood ratio criterion is

$$\Phi = \frac{\sup_{\Theta_o} L(\lambda_1, \lambda_2 \mid \underline{x}, \underline{y})}{\sup_{\Theta} L(\lambda_1, \lambda_2 \mid \underline{x}, \underline{y})}$$

$$= \frac{\left(\frac{S_{r_1}}{T_{r_2}}\right)^{r_1}}{\left(1 + k \frac{S_{r_1}}{T_{r_2}}\right)^{r_1 + r_2}} .$$

Thus, the critical region is

$$w = \left\{ k \frac{S_{r_1}}{T_{r_2}} < k_1 \text{ or } k \frac{S_{r_1}}{T_{r_2}} > k_2 \right\} ,$$

where k_1 and k_2 are determined so that

$$w = \left[(\underline{x}, \underline{y}) : \left\{ 0 < \frac{S_{r_1}}{T_{r_2}} < \frac{k r_1}{r_2} F_{1 - \frac{\alpha}{2}}(2r_1, 2r_2) \right\} \cup \left\{ \frac{k r_1}{r_2} F_{\frac{\alpha}{2}}(2r_1, 2r_2) < \frac{S_{r_1}}{T_{r_2}} < \infty \right\} \right] .$$

If z_3 be the p-value of the test, then the shrinkage estimator of 'P' is given by

$$\hat{\hat{P}}_{II(z_3)} = (1 - z_3) \hat{P}_{II} + z_3 P_o . \tag{3.4}$$

For $\tilde{P}_{II}$ given in (1.8) , we define the shrinkage estimator of 'P' to be

$$\tilde{\tilde{P}}_{II} = \alpha_{10} \tilde{P}_{II} + (1 - \alpha_{10}) P_o , 0 \leq \alpha_{10} \leq 1 . \tag{3.5}$$

The value of α_{10} , which minimizes the MSE of $\tilde{\tilde{P}}_{II}$ is given by

$$\alpha_{10} = \frac{[P - P_o][E(\tilde{P}_{II}) - P_o]}{[E(\tilde{P}_{II})^2 - 2P_o E(\tilde{P}_{II}) + P_o^2]} . \tag{3.6}$$

Denoting by

$$a_i = (-1)^i (r_2 - 1) \binom{r_2 - 2}{i} B(i+1, r_1) \quad \text{and}$$

$$b_i = (-1)^i (r_2 - 1) \binom{r_1 - 1}{i} B(i+1, r_2 - 1),$$

from (1.8),

$$E(\tilde{P}_{II}^2) = \sum_{i=0}^{r_2-2} \sum_{j=0}^{r_2-2} a_i a_j E\left\{ \left(\frac{S_{r_1}}{T_{r_2}} \right)^{i+j+2} I(S_{r_1} < T_{r_2}) \right\} + \sum_{i=0}^{r_1-1} \sum_{j=0}^{r_1-1} b_i b_j E\left\{ \left(\frac{T_{r_2}}{S_r} \right)^{i+j} I(T_{r_2} < S_{r_1}) \right\}. \tag{3.7}$$

We have,

$$E\left\{ \left(\frac{S_{r_1}}{T_{r_2}} \right)^{i+j+2} I(S_{r_1} < T_{r_2}) \right\} = \left(\frac{\lambda_2}{\lambda_1} \right)^{i+j+2} \left(\frac{r_1}{r_2} \right)^{r_1+i+j+2} \int_0^1 \frac{F^{r_1+i+j+1}}{B(r_1, r_2) \left(1 + \frac{r_1}{r_2} F \right)^{r_1+r_2}} dF. \tag{3.8}$$

Similarly,

$$E\left\{ \left(\frac{T_{r_2}}{S_{r_1}} \right)^{i+j} I(T_{r_2} < S_{r_1}) \right\} = \left(\frac{\lambda_1}{\lambda_2} \right)^{i+j} \left(\frac{r_2}{r_1} \right)^{r_2+i+j} \int_0^1 \frac{F^{r_2+i+j-1}}{B(r_2, r_1) \left(1 + \frac{r_2}{r_1} F \right)^{r_1+r_2}} dF. \tag{3.9}$$

Since λ_1 and λ_2 are unknown, they are estimated by their UMVUES.

Based on the p-value z_3 already defined, the shrinkage estimator of 'P' is given by

$$\tilde{\tilde{P}}_{II(z_3)} = (1 - z_3) \tilde{P}_{II} + z_3 P_o. \tag{3.10}$$

Under Type I censoring, the MLE of 'P' is given by

$$\hat{P}_I = \frac{\hat{\lambda}_{1I}}{\hat{\lambda}_{1I} + \hat{\lambda}_{2I}},$$

and the UMVUE of 'P' is

$$\tilde{P}_I = \begin{cases} r_2 \sum_{i=0}^{r_1} (-1)^i \binom{r_1}{i} \left(\frac{m}{n} \right)^i B(i+1, r_2), & m < n \\ r_2 \sum_{i=0}^{r_2-1} (-1)^i \binom{r_2 - 1}{i} \left(\frac{n}{m} \right)^{i+1} B(i+1, r_1+1), & n < m. \end{cases}$$

We observe that the estimators of 'P' can be presented as the ratio of two Poisson rv's. Therefore, the distributions of the estimators cannot be obtained. We, therefore, conduct simulation study for the shrinkage estimators of 'P' under type I censoring. The results are presented in Table 3.2.

4. Numerical Findings and Conclusion

A simulation study is conducted to investigate the performance of the above estimators.

The indices of our simulations for Section 2 are:

λ : the true value of the parameter and is taken to be 1

λ_o : prior guess value of λ and is taken to be 0.50, 0.80, 1.00, 1.20, 1.50 and 2.00

q : power of λ and is taken to be 1, 2 and 3

CP: the censoring proportion and is taken to be 0.25, 0.50 and 0.75

t_o: truncation time point and taken as 0.40, 0.80 and 1.00

For each combination of λ and λ_o, 1000 samples of size 40 were generated from the distribution given in (1.1). The shrinkage estimators for λ^q are calculated under both type II and type I censorings (considering the above values of CP and t_o respectively) and the relative efficiencies of these estimators to the corresponding maximum likelihood estimators are calculated as the ratio of the mean squared error of the MLE to the mean squared error of the shrinkage estimator. Similarly, the relative efficiencies of these estimators to the UMVUES are computed. Table 1.1 presents the relative efficiencies of the shrinkage estimators of λ^q for λ =1, under type II censoring. Similar results for type I censoring are presented in table 1.4.

Under type II censoring, we can observe that $\hat{\hat{\lambda}}_{II}^q$ performs the best, followed by $\hat{\hat{\lambda}}_{II(z_1)}^q$ and $\tilde{\tilde{\lambda}}_{II(z_1)}^q$ is the worst estimator. One point to be noted is that $\tilde{\tilde{\lambda}}_{II}^q$ is equally efficient as $\tilde{\lambda}_{II}^q$ (It can be seen from the formula also). Also, as q increases, the relative efficiencies of the estimator $\hat{\hat{\lambda}}_{II}^q$ increases when λ=λ_o.

Under type I censoring, for q=1, $\hat{\hat{\lambda}}_I^q$ and $\tilde{\tilde{\lambda}}_I^q$ are equally efficient as can be seen from the formula (as $\hat{\lambda}_I^q = \tilde{\lambda}_I^q$ for q=1). However, for q=2 and q=3, the estimator $\tilde{\tilde{\lambda}}_I^q$ performs better than $\hat{\hat{\lambda}}_I^q$ except when λ_o=0.5.

Also, we can observe that the shrinkage estimators under both type I and type II censorings seem to perform better for small sample sizes than for large sample sizes when λ= λ_o.

The indices of our simulations for Section 4 are:

P : the true value of P=P(X>Y) and is taken to be 0.65,0.70 and 0.80

P_o: the initial estimate of 'P' and is taken to be
0.55, 0.60, 0.65, 0.70, 0.75 and 0.80 when P=0.65
0.60, 0.65, 0.70, 0.75, 0.80 and 0.85 when P=0.70
0.70, 0.75, 0.80, 0.85, 0.90 and 0.95 when P=0.80

r_1: number of X observations and is taken to be 20 and 30

r_2: number of Y observations and is taken to be 20 and 30

For each combination of P and P_o , 1000 samples of size 40 were generated for X from the distribution given in (1.1), taking λ_1 = 1 and 1000 samples of size 40 were generated for Y from the same distribution with $\lambda_2 = \frac{1}{P} - 1$. The shrinkage estimators for 'P' are calculated under type II censoring and their relative efficiencies are computed.

Table 3 presents the relative efficiencies of the shrinkage estimators for 'P' under type II censoring.

From the tables, we can observe that $\tilde{\tilde{P}}_{II}$ has the highest relative efficiency. The shrinkage estimators can be arranged in terms of overall performance as follows (from best to worst); $\tilde{\tilde{P}}_{II}$ - $\tilde{\tilde{P}}_{II(z_3)}$ - $\hat{\hat{P}}_{II}$ - $\hat{\hat{P}}_{II(z_3)}$.

Under Type I censoring, the shrinkage estimators for 'P' are directly calculated using simulation. Table 3.2 gives the respective relative efficiencies.

From the tables, we observe that $\hat{\hat{P}}_{I}$ performs better than $\tilde{\tilde{P}}_{I}$ when $P = P_o$ or when P is close to P_o

Table 1.1: Relative efficiencies of the estimators for λ^q (when λ=1) under Type II censoring

CP	λ_0	EF1	EF2	EF3	EF4	EF1	EF2	EF3	EF4	EF1	EF2	EF3	EF4
		q = 1				q = 2				q = 3			
0.25	0.50	0.4663	0.8501	1.0000	0.4914	0.4388	0.9096	1.0000	0.3565	0.6044	0.9613	1.0000	0.2522
0.25	0.80	0.9302	0.8740	1.0000	0.5644	1.1154	0.9260	1.0000	0.4380	1.6824	1.0051	1.0000	0.3291
0.25	1.00	2.1722	1.4783	1.0000	0.9982	3.8702	1.6013	1.0000	0.9360	10.9352	1.5989	1.0000	0.8472
0.25	1.20	0.8577	1.0785	1.0000	0.7831	0.7520	1.1364	1.0000	0.6873	0.7772	1.2496	1.0000	0.5748
0.25	1.50	0.5887	0.8974	1.0000	0.6087	0.3226	0.8571	1.0000	0.4468	0.1438	0.8695	1.0000	0.2951
0.25	2.00	0.6668	0.8574	1.0000	0.5125	0.4025	0.7182	1.0000	0.3157	0.1385	0.5768	1.0000	0.1668
0.50	0.50	0.8037	0.8894	1.0000	0.6474	1.1993	0.9512	1.0000	0.7583	2.5446	0.9845	1.0000	0.8627
0.50	0.80	1.4578	1.0835	1.0000	0.8335	2.0407	1.0474	1.0000	0.9022	4.2071	1.0232	1.0000	0.9597
0.50	1.00	1.7210	1.1889	1.0000	0.8590	2.4041	1.1525	1.0000	0.9442	5.0746	1.0990	1.0000	1.0096
0.50	1.20	1.3431	1.0784	1.0000	0.7516	1.7213	1.1086	1.0000	0.8440	3.0446	1.1272	1.0000	0.9393
0.50	1.50	1.0534	1.0439	1.0000	0.7183	1.1730	1.1275	1.0000	0.8110	1.4953	1.2475	1.0000	0.9071
0.50	2.00	0.8669	0.9369	1.0000	0.6476	0.7662	0.8804	1.0000	0.6185	0.6536	0.8352	1.0000	0.5193
0.75	0.50	0.6076	0.9221	1.0000	0.3729	0.6219	0.9504	1.0000	0.3737	0.6966	0.9706	1.0000	0.3842
0.75	0.80	1.1156	1.0240	1.0000	0.4595	1.1562	1.0070	1.0000	0.4549	1.3029	0.9980	1.0000	0.4614
0.75	1.00	1.8140	1.2967	1.0000	0.5262	1.9001	1.3165	1.0000	0.5284	2.2944	1.3179	1.0000	0.5468
0.75	1.20	1.2466	1.0557	1.0000	0.4196	1.1537	1.0769	1.0000	0.4135	1.1713	1.1102	1.0000	0.4216
0.75	1.50	0.9739	0.9675	1.0000	0.3854	0.8799	0.9422	1.0000	0.3640	0.7975	0.9084	1.0000	0.3465
0.75	2.00	0.9533	0.9868	1.0000	0.3936	0.9229	0.9648	1.0000	0.3767	0.8316	0.9138	1.0000	0.3631

EF1 denotes the relative efficiency of $\hat{\hat{\lambda}}^q_{II}$ with respect to $\hat{\lambda}^q_{II}$.

EF2 denotes the relative efficiency of $\hat{\hat{\lambda}}^q_{II(z_3)}$ with respect to $\hat{\lambda}^q_{II}$.

EF3 denotes the relative efficiency of $\tilde{\tilde{\lambda}}^q_{II}$ with respect to $\tilde{\lambda}^q_{II}$.

EF4 denotes the relative efficiency of $\tilde{\tilde{\lambda}}^q_{II(z_3)}$ with respect to $\tilde{\lambda}^q_{II}$.

Table 1.4: Relative efficiencies of the estimators for λ^q (when λ=1) under Type I censoring

t_0	λ_0	EF1	EF2	EF1	EF2	EF1	EF2
		q = 1		q = 2		q = 3	
0.40	0.50	0.8462	0.8462	1.0769	0.8932	1.6951	1.0320
0.40	0.80	1.5600	1.5600	1.8881	1.7981	2.9968	2.0668
0.40	1.00	2.0253	2.0253	2.3969	2.5534	3.9860	3.1883
0.40	1.20	1.5248	1.5248	1.4793	1.6101	1.7996	1.6785
0.40	1.50	0.9605	0.9605	0.7456	0.7842	0.6446	0.6301
0.40	2.00	0.8243	0.8243	0.6365	0.6130	0.5067	0.4497
0.50	0.50	0.7376	0.7376	0.7372	0.6992	0.8057	0.7288
0.50	0.80	1.3891	1.3891	1.3414	1.3857	1.4850	1.3777
0.50	1.00	1.7088	1.7088	1.6534	1.8216	1.9834	1.8663
0.50	1.20	1.4005	1.4005	1.1718	1.3712	1.1138	1.3023
0.50	1.50	1.1201	1.1201	0.8400	0.9742	0.6613	0.8305
0.50	2.00	1.0709	1.0709	0.8804	0.9430	0.8201	0.8325
0.80	0.50	0.7821	0.7821	0.7623	0.7611	0.7518	0.7849
0.80	0.80	1.3583	1.3583	1.2303	1.3217	1.1843	1.2616
0.80	1.00	1.4027	1.4027	1.2402	1.3950	1.2041	1.3330
0.80	1.20	1.2785	1.2785	1.0950	1.2485	1.0294	1.1868
0.80	1.50	1.1847	1.1847	1.0188	1.1419	0.9914	1.0904
0.80	2.00	1.1180	1.1180	0.9993	1.0751	0.9999	1.0393

EF1 denotes the relative efficiency of $\hat{\hat{\lambda}}^q_{I}$ with respect to $\hat{\lambda}^q_{I}$.

EF2 denotes the relative efficiency of $\tilde{\tilde{\lambda}}^q_{I}$ with respect to $\tilde{\lambda}^q_{I}$.

Table 3.1 : Relative efficiencies of the estimators for 'P' under Type II censoring

r_1	r_2	P_0	EF1	EF2	EF3	EF4	P_0	EF1	EF2	EF3	EF4
			P = 0.65					P = 0.70			
20	20	0.55	0.8747	0.8935	8.6366	1.3112	0.60	1.12935	0.99823	18.60052	1.11717
20	20	0.60	1.3768	1.1896	33.1866	2.1387	0.65	1.34096	1.10779	65.09544	1.42777
20	20	0.65	2.0898	1.4682	12259.9	4.1912	0.70	1.65297	1.29699	3157.477	4.48792
20	20	0.70	1.4433	1.1278	40.7103	3.7192	0.75	1.48301	1.25862	126.3240	2.50296
20	20	0.75	1.0065	0.9074	9.7796	1.8742	0.80	0.88466	0.90377	26.76857	2.65255
20	20	0.80	0.9139	0.9121	4.3181	1.2077	0.85	0.68749	0.84600	11.39580	1.33798
20	30	0.55	0.8367	0.8525	4.5872	1.2575	0.60	1.19634	0.99385	1.02817	1.12319
20	30	0.60	1.4188	1.2145	3.6289	1.9642	0.65	1.45046	1.15393	1.04777	1.51661
20	30	0.65	2.4049	1.5800	2.3518	3.8498	0.70	1.92959	1.48651	1.15393	5.22321
20	30	0.70	1.4160	1.1090	1.5529	3.4440	0.75	1.25041	1.16939	1.00841	3.31816
20	30	0.75	0.9174	0.8692	1.2145	1.7185	0.80	0.66835	0.79089	1.00289	1.89557
20	30	0.80	0.8570	0.9150	1.0541	1.1416	0.85	0.60325	0.87574	1.00077	1.11074
30	20	0.55	0.9389	0.9215	9.9706	1.1710	0.60	1.09220	0.99787	21.44579	1.05095
30	20	0.60	1.3246	1.1332	39.8816	1.6750	0.65	1.21835	1.04993	85.77745	1.22399
30	20	0.65	1.9621	1.4208	5.8E+10	3.1912	0.70	1.41625	1.16233	1.6.E+10	3.56003
30	20	0.70	1.3457	1.1185	39.8856	3.9938	0.75	1.39624	1.19633	85.80269	2.97642
30	20	0.75	0.8683	0.8636	9.9712	2.1097	0.80	0.95976	0.96719	21.44926	1.85767
30	20	0.80	0.7645	0.8740	4.4316	1.2608	0.85	0.71420	0.85612	9.53284	1.43372
30	30	0.55	0.9074	0.8791	10.0304	1.1329	0.60	1.16377	0.99873	19.34240	1.04655
30	30	0.60	1.3243	1.1172	39.0724	1.5483	0.65	1.29019	1.05997	71.87650	1.24242
30	30	0.65	2.1038	1.4825	35761.8	2.8128	0.70	1.54244	1.23926	11374.45	4.70741
30	30	0.70	1.3361	1.1277	44.5385	3.5573	0.75	1.31327	1.19363	101.7306	2.23304
30	30	0.75	0.8067	0.8420	10.8464	1.9089	0.80	0.76185	0.85556	23.42691	2.13786
30	30	0.80	0.7249	0.8821	4.7969	1.1851	0.85	0.65270	0.88676	10.18862	1.14824

EF1 denotes the relative efficiency of $\hat{\hat{P}}_{II}$ with respect to $\hat{P}_{II}$.

EF2 denotes the relative efficiency of $\hat{\hat{P}}_{II(z_1)}$ with respect to $\hat{P}_{II}$.

EF3 denotes the relative efficiency of $\tilde{\tilde{P}}_{II}$ with respect to $\tilde{P}_{II}$.

EF4 denotes the relative efficiency of $\tilde{\tilde{P}}_{II(z_2)}$ with respect to $\tilde{P}_{II}$.

Table 3.2 : Relative efficiencies of the estimators for 'P' under Type I censoring

t_{ox}	t_{oy}	P_0	EF1	EF2	P_0	EF1	EF2	P_0	EF1	EF2
			P = 0.65			P = 0.70			P=0.80	
0.4	0.4	0.55	2.0529	12.3154	0.60	3.2551	8.9529	0.70	1.9608	1.0020
0.4	0.4	0.60	8.1537	49.1242	0.65	13.1623	35.7842	0.75	7.8152	1.0030
0.4	0.4	0.65	35883	14113	0.70	12986	197261	0.80	43926	1.0076
0.4	0.4	0.70	8.2267	48.6763	0.75	12.8310	35.6239	0.85	7.8901	1.0048
0.4	0.4	0.75	2.0493	12.2235	0.80	3.2305	8.9113	0.90	1.9623	1.0063
0.4	0.4	0.80	0.9127	0.9999	0.85	1.4384	3.9606	0.95	0.8726	1.0039
0.4	0.8	0.55	1.6063	6.4291	0.60	1.1760	8.9529	0.70	2.1921	1.0020
0.4	0.8	0.60	6.4072	25.7193	0.65	4.6860	35.7842	0.75	8.7648	1.0030
0.4	0.8	0.65	43077	40734	0.70	132840	197261	0.80	316410	1.0076
0.4	0.8	0.70	6.4558	25.4752	0.75	4.6978	35.6239	0.85	8.7820	1.0048
0.4	0.8	0.75	1.6055	6.3789	0.80	1.1764	8.9113	0.90	2.1939	1.0033
0.4	0.8	0.80	0.7139	2.8361	0.85	0.5235	3.9606	0.95	0.9743	1.0011
0.8	0.4	0.55	1.8396	1.0010	0.60	2.7545	19.0484	0.70	1.8469	1.0087
0.8	0.4	0.60	7.3271	1.0019	0.65	11.1233	39.9222	0.75	7.3714	1.0099
0.8	0.4	0.65	67592	1.0081	0.70	14907	53.1232	0.80	103689	1.0142
0.8	0.4	0.70	7.3736	1.0048	0.75	10.8990	25.4239	0.85	7.4192	1.0132
0.8	0.4	0.75	1.8340	1.0040	0.80	2.7401	7.9321	0.90	1.8466	1.0121
0.8	0.4	0.80	0.8163	1.0034	0.85	1.2195	3.3211	0.95	0.8210	1.0110
0.8	0.8	0.55	1.4602	14.9805	0.60	0.9504	19.0484	0.70	2.1815	1.0087
0.8	0.8	0.60	5.8364	59.9222	0.65	3.7938	39.9222	0.75	8.7251	1.0099
0.8	0.8	0.65	297563	64.1232	0.70	247028	53.1232	0.80	3452333	1.0142
0.8	0.8	0.70	5.8561	35.6239	0.75	3.7949	25.4239	0.85	8.7319	1.0132
0.8	0.8	0.75	1.4587	8.9113	0.80	0.9495	7.9321	0.90	2.1825	1.0121
0.8	0.8	0.80	0.6484	3.9606	0.85	0.4224	3.3211	0.95	0.9694	1.0110

EF1 denotes the relative efficiency of $\hat{\hat{P}}_{I}$ with respect to $\hat{P}_{I}$.

EF2 denotes the relative efficiency of $\tilde{\tilde{P}}_{I}$ with respect to $\tilde{P}_{I}$.

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