Special Issue of International Journal of Linguistics and Computational Applications ISSN 2394-6393 (Online) ISSN 2394-6385 (Print)
Proceedings of the UGC Sponsored - National Conference on "Recent Statistical Computing Techniques and their Applications"
Department of Statistics and Department of Computer Science, Ramanujan College,University of Delhi, Delhi.
March 11-12, 2016. **Published on January 2017**

Sensitivity Analysis of Sequential Predictive Normal Testing Procedure

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Abstract— Many studies concern with the robustness of character of many kind of acceptance sampling plans. Many such studies also analyze the robustness of character of sequential testing procedures when the given failure time distribution has a monotone failure rate. A wide literature on the life testing plans in the Bayesian Statistics is also available where updating prior with experimental data has been the main concern. Emphasizing the point that the initial normal lifetime distribution can also be upgrade in terms of prior variations in the contain parameters. The current paper deals with the analysis of the robustness of sequential normal testing procedures when the parameter (mean) of the initial normal distribution is treated as a random variable. The robustness of the Coefficient of variation of the random variable n, in view of prior variations, is also analyzed.
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Keywords— Robustness; SNTP; OC; ASN and Coefficient of variation function.

1. Introduction

Themainrole in the field of sequential test of statistical hypothesis is due to A. Wald (1947), who developed sequential probability ratio test (SPRT) for testing a simple hypothesis v/s a simple alternative. Here we find the formula for operating characteristic (O.C.) and average sample numbers (ASN) function of the SPRT. The robust character of the SPRT, when the given distribution has suffered a change, has been studied by many researcher concerning with various probabilistic models, like Montagne and Singapurwala (1985), Sharma K.K. and Rana (1990), Sharma K.K. and Bhutani (1992) and others. The robust character of these procedures has been analyzedin terms of producer's and consumer's risk and the ASN.

Further on using different life time models, the studies in [2A] involve a wide literature on the life testing plans in Bayesian statistics in which upgrading the prior with experimental data has been the main concern. However, in the Bayesian statistics, it should be acknowledged that priors do have an impression on the initial life time distribution and thus the initial distribution can be upgraded in terms of prior variations. This compound distribution be used in the analysis. In this paper we involve SPRT for testing the $H_0 : \mu = \mu_0$ v/s $H_1 : \mu = \mu_1$ when samples are sequentially taken from $N(\mu, \sigma^2), \sigma^2$ being known. Further, when we takeμ to be a random variable we use the prior of μ to upgrade the initial life time distribution, $N(\mu, \sigma^2)$ and use this compound distribution to analyze the robustness of SPRT when μ is taken to be a random variable. The concept has been Emphasized in the present paper. Further, it was also noted that the initial life time distribution can be further upgraded in terms of the compound distribution. This further upgraded distribution is also termed as predictive basic distribution.

In respect of given above, the present study involves the robustness of SPRT, when predictive life time distribution is used in the analysis. A comparison in respect of OC, ASN curve and C.V curve in the given conditions has been used as the basis of analysis.

2. Notations

N(μ, σ2)	:	Normal distribution with mean μ and variance σ2
p.d.f.	:	Probability density function
SPRT	:	Sequential probability ratio test
SNTP	:	Sequential normal testing procedure
L(μ)	:	The OC function, the probability of accepting H0 when μ is the true parametric value
n	:	The sample size needed to terminate the sequential testing. Thus n is a random variable.
Eμ(n)	:	ASN functions for fixed μ.
Vμ(n)	:	Variance of the random variable n for fixed μ
CVμ(n)	:	Co-efficient of variation of the random variable n for fixed μ
α	:	Size of the type I error, also called producer's risk in quality control terminology.
β MTSF	: :	Size of the type II error, also called consumer's risk in quality control terminology. Mean Time to system failure

3. Statistical Background

For developing the procedure we assume that (i)The failure time distribution of X is $N(\mu, \sigma_1^2)$ with, p.d $f_1\left(x,\mu,\sigma_1^2\right)=\frac{1}{\sigma_1\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma_1}\right)^2};\ -\infty<x<\infty,\ \sigma_1^2>0$ (1)

Here , μ is the MTSF and σ1 is assumed known.

(ii) The lot to lot quality, μ is a random variable having its known conjugate prior distribution as normal with p.d.f.

$$g\left(\mu,\theta,\sigma_2^2\right)=\frac{1}{\sigma_2\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{\mu-\theta}{\sigma_2}\right)^2};\ -\infty<\mu<\infty,\ \sigma_2^2>0 \quad (2)$$

(iii) Further let $\underset{\sim}{x}=(x_1,x_2,\ldots\ldots\ldots\ldots,x_n)$ be random sample of size n from the population in (1) then the posterior distribution of μ forgiven X~ will be $\pi\left(\frac{\mu}{\underset{\sim}{x}}\right)=\sqrt{\frac{n\sigma_2^2+\sigma_1^2}{\sigma_1^2\sigma_2^2}}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}\left[\mu-\frac{n\bar{x}\sigma_2^2+\theta\sigma_1^2}{\sigma_1^2+n\sigma_2^2}\right]^2\Big/\left(\frac{\sigma_1^2+n\sigma_2^2}{\sigma_1^2\sigma_2^2}\right)}$

$$\Rightarrow \pi\left(\frac{\mu}{\underset{\sim}{x}}\right)\sim N\left(\lambda,\xi^2\right) \quad (3)$$

where,

$$\lambda=\frac{n\bar{x}\sigma_2^2+\theta\sigma_1^2}{n\sigma_2^2+\sigma_1^2} \quad \text{and} \quad \xi^2=\frac{\sigma_1^2\sigma_2^2}{n\sigma_2^2+\sigma_1^2}$$

(iv) Finally, in view of (1) and (3) the predictive distribution of the future observations [8] that is the updated compound distribution of X can also be obtained as $h(x/\underset{\sim}{x})=\int_{-\infty}^{\infty}f(x,\mu,\sigma_1^2)\prod(\mu/\underset{\sim}{x})\,d\mu$

$$\Rightarrow \frac{1}{\sqrt{2\pi}\left(\sqrt{\xi^2+\sigma_1^2}\right)}e^{-\frac{1}{2}\left[\frac{x-\lambda}{\sqrt{\xi^2+\sigma_1^2}}\right]^2};$$

$$\Rightarrow h\left(x/\underset{\sim}{x}\right)\sim N\left(\lambda,\beta^2\right)=f_3\left(x,\lambda,\beta^2\right) \quad (4)$$

Now here

$$E(X)=\left[\lambda=\frac{n\bar{x}\sigma_2^2+\theta\sigma_1^2}{n\sigma_2^2+\sigma_1^2}\right] \text{ and}$$

$$Var(X)=\left[\beta^2=\sigma_1^2+\xi^2\right]$$

Also the distribution of x in (4) is $N(\lambda,\beta^2)$.The respective basic and predictive distributions in (1) and (4) enable us to analyse the robust character of SNTP with respect to α , β and Eμ (n) in the following situations-

- The basic distribution in (1) is used in the analysis when μ is considered as a constant.
- The predictive basic distribution in (4) which is updated in respect of posterior distribution of μ is used in the analysiswhen μ is considered as a random variable. Obviously, this updated distribution also includes information on experimental data.

4. OC and ASN Functions for SNTP when μ is Considered as a Constant

In this case, SNTP for testing the null hypothesis $H_0:\mu=\mu_0=\lambda_0$ v/s $H_1:\mu=\mu_1=\lambda_1$ we assume that the sample observations are being sequentially recorded from the basic distribution in (1) i.e. $N(\mu,\sigma_1^2),\sigma_1^2$ being known. Here, following [13]. The approximations to the OC function of the present SNTP can be obtained by considering the values of $g=g(\mu)\neq 0$ which satisfy

$$E\left[\frac{f_1(x,\mu_1,\sigma_1)}{f_1(x,\mu_0,\sigma_1)}\right]^g=1$$

or $\int_{-\infty}^{\infty}\left[\frac{f_1(x,\mu_1,\sigma_1)}{f_1(x,\mu_0,\sigma_1)}\right]^g f_1(x,\mu,\sigma_1)\,dx=1$ (5)

On simplifying (5), one gets the following pair of parametric equations providing approximations to the OC function---

$$\left[g=\frac{(\mu_0+\mu_1)-2\mu}{(\mu_1-\mu_0)}\right] \text{ and}$$

$$\left[L(\mu)=\frac{A^g-1}{A^g-B^g}\right] \quad \ldots\ldots(6)$$

A plot of $L(\mu)$ against μ provides us the OC curve. Here,

$$A=\frac{1-\beta}{\alpha},\qquad B=\frac{\beta}{1-\alpha} \text{ and}$$

$$\mu_1\neq\mu_0$$

Further, the approximated formula for ASN of present SNTP will be

$$E_\mu(n)=\frac{L(\mu)LogB+(1-L(\mu))LogA}{E_\mu(z)} \quad \ldots(4)$$

$$E_\mu(n)=\frac{L(\mu)LogB+(1-L(\mu))LogA}{\frac{1}{2\sigma_1^2}\left[\left(\mu_0^2-\mu_1^2\right)+2\mu(\mu_1-\mu_0)\right]} \quad (7)$$

Here $E_\mu(z)=\frac{1}{2\sigma_1^2}\left[\left(\mu_0^2-\mu_1^2\right)+2\mu(\mu_1-\mu_0)\right]$ (8)

The notable features of the equation in (6) is that this is independent of σ_1^2. Thus, the OC curves for the present SNTP remains static with variations in σ_1^2. As a consequence the points $[\mu_0,(1-\alpha)]$ and (μ_1,β) on the OC curve also remain static with variations in σ_1^2 as such. The present SNTP is found to be robust in respect to 'α' and 'β' with variation in σ_1^2. On the other hand, the expression for $E_\mu(n)$ in (7) is a function of σ_1^2. However, it seems logical that variation in σ_1^2 should obviously have an effect on the size of two errors, but the present SNTP is observed to be an exception in this regard. Still, the results in (6) and (7) provide us a basis for investigating the robustness of SNTP in respect of α, β and ASN when μ is considered as a random variable.

5. Robustness of the Present SNTP in Respect of Prior Variations with Experimental Data

Given a sequence of observation $x_1, x_2, \ldots\ldots\ldots, x_n$ from (1) Suppose one wishes to test the null hypothesis $H_0 : \mu = \mu_0 = \lambda_0$ v/s H1 : $\mu = \mu_1 (> \mu_0) = \lambda_1$,In order to study the robustness of the SPRT developed in section 4 in respect of OC and ASN function. We consider h1 as solution of the equation

$$E\left[\int_{-\infty}^{\infty}\left\{\frac{f_3(x,\lambda_1,\beta)}{f_3(x,\lambda_0,\beta)}\right\}\right]^{h^1} = 1$$

$$\int_{-\infty}^{\infty}\left[\frac{f_3(x,\lambda_1,\beta)}{f_3(x,\lambda_0,\beta)}\right]^{h^1} f_3(x,\lambda,\beta)\,dx = 1 \tag{9}$$

It is notable that the expression in (9) is in respect of the predictive distribution in (3). The OC curve for testing $H_0 : \mu = \mu_0 = \lambda_0$ v/s $H_1 : \mu = \mu_1 = \lambda_1$ is obtained approximately by the following pair of equations on solving (9), we get

$$\left[h' = \frac{(\lambda_1 + \lambda_0) - 2\lambda}{(\lambda_1 - \lambda_0)}\right]$$

and

$$L(\lambda) = \frac{A^{h'} - 1}{A^{h'} - B^{h'}} \tag{10}$$

Thus a comparison of the OC curve in (5) and (10) help us in analyzing the robust character of SNTP in respect of α and β when updated prior has been used on using experimental data.

Similarly, for analyzing the robustness of ASN function for SNTP we obtain

$$E_\lambda(z) = E\left[Log\,\frac{f_3(x,\lambda_1,\beta)}{f_3(x,\lambda_0,\beta)}\right]$$

$$= \int_{-\infty}^{\infty}\left(Log\,\frac{f_3(x,\lambda_1,\beta)}{f_3(x,\lambda_0,\beta)}\right) f_3(x,\lambda,\beta)\,dx$$

$$\Rightarrow E_\lambda(z) = \frac{1}{2\beta^2}\left[\left(\lambda_0^2 - \lambda_1^2\right) + 2\lambda(\lambda_1 - \lambda_0)\right] \tag{11}$$

Here, also the expectation is taken in respect of the predictive normal distribution on using (9) one easily gets,

$$E_\lambda(n) = \frac{L(\lambda)\,LogB + (1 - L(\lambda))\,Log\,A}{\frac{1}{2\beta^2}\left[\left(\lambda_0^2 - \lambda_1^2\right) + 2\lambda(\lambda_1 - \lambda_0)\right]} \tag{12}$$

For σ1 = β, Thus a comparison of the ASN in (7) and in (12) help us in analyzing the robust character of SNTP in respect of ASN when the prior can be updated in the form of posterior on using experimental data in Bayesian framework.

6. Analyzing the Robustness of the Consistency of n when μ is considered as a Random Variable

Recognizing the fact that in all sequential testing procedures n is a random variable, as such, the ASN function, i.e., $E_\mu(n)$ alone cannot be taken as a measure of effectiveness of the testing procedure. In this regard, $V_\mu(n)$ is a measure of the consistent behavior which can be analyzed by using a relative measure of dispersion, called the co-efficient of variation (C.V.) the C.V. in the present case will be

$$C.V_\mu(n) = \frac{\sqrt{V_\mu(n)}}{E_\mu(n)} \times 100 \tag{13}$$

For developing formula for $C.V_\mu(n)$ we proceed to develop an approximate formula for $V_\mu(n)$ as under. Following the assumptions as used in the derivation of $E_\theta(n)$ by Wald (1947), we consider

$$(Z_1 + Z_2 + \ldots.. Z_N) = (Z_1 + Z_2 + \ldots.. Z_n) + (Z_{n+1} + \ldots.. Z_N)$$

Where

$$Z_\alpha = \left[Log\,\frac{f_2(x_\alpha,\theta_1,\sigma^2)}{f_2(x_\alpha,\theta_0,\sigma^2)}\right]$$

Also for α > n, the random variable Z_α is distributed independently of n.

Thus $V(Z_1 + Z_2 + \ldots.. Z_N) = V(Z_1 + Z_2 + \ldots.. Z_n) + V(Z_{n+1} + \ldots.. Z_N)$ (14)

Now, consider the term

$$V\left(\sum_{i=n+1}^{N} Z_i\right) = V\left(\sum_{i=n+1}^{N} \frac{Z_i}{n}\right)$$

$$= V\left[E\left(\sum_{i=n+1}^{N}\frac{Z_i}{n}\right)\right] + E\left[V\left(\sum_{i=n+1}^{N}\frac{Z_i}{n}\right)\right]$$

$$= V\left[(N-n)E(z)\right] + E\left[(N-n)V(z)\right]$$

$$= \left[E(z)\right]^2 V(n) + V(z)E(N-n)$$

$$= \left[E(z)\right]^2 V(n) + V(z)\left[N - E(n)\right] \tag{15}$$

On using (14) in (13), One gets

$$V\left(\sum_{i=1}^{n} Z_i\right)+\left[E(z)\right]^2 V(n)-V(z)E(n)=0$$

or

$$V(n)=\frac{V(z)E(n)-V\left(\sum_{i=1}^{n} Z_i\right)}{\left[E(z)\right]^2}$$

Since all the terms in $V(n)$ are function of μ, thus the above can be re-written as

$$V_\mu(n)=\frac{V_\mu(z)E_\mu(n)-V_\mu\left(\sum_{i=1}^{n} Z_i\right)}{\left[E_\mu(z)\right]^2} \quad (16)$$

6.1 $C.V_\mu(n)$ when μ is Constant

In this case, the expression for $E_\mu(n)$ and $E_\mu(z)$ are as given in (7) and (8) respectively, further, it is easy to obtain

$$V_\mu(z)=E_\mu\left(z^2\right)-\left[E_\mu(z)\right]^2$$

$$=\frac{(\mu_1-\mu_0)^2}{\sigma_1^2} \quad (17)$$

and

$$V\left(\sum_{i=1}^{n} Z_i\right)=L(\mu)\left[1-L(\mu)\right]\left[LogB-LogA\right]^2 \quad (18)$$

On using (7), (8) (17) and (18) in (16), one gets $V_\mu(n)$ in the present case and therefore,

$$CV_\mu(n)=\frac{\sqrt{V_\mu(n)}}{E_\mu(n)}\times 100 \quad (19)$$

6.2 $C.V._\lambda(n)$ when μ is random variable

In this case, the expression for $E_\lambda(n)$ and $E_\lambda(z)$ are as given in (12) and (11) respectively, further one easily gets

$$V_\lambda(z)=E_\lambda\left(z^2\right)-\left[E_\lambda(z)\right]^2=\frac{(\lambda_1-\lambda_0)^2}{\beta^2} \quad (20)$$

and

$$V\left(\sum_{i=1}^{n} z_i\right)=L(\lambda)\left[1-L(\lambda)\right]\left\{LogB-LogA\right\}^2 \quad (21)$$

On using (11), (12), (20) and (21) in (16) one easily get $V_\lambda(n)$ and consequently

$$C.V_\lambda(n)=\frac{\sqrt{V_\lambda(n)}}{E_\lambda(n)}\times 100 \quad (22)$$

Now, the trend in $C.V_\lambda(n)$ and $C.V_\mu(n)$ for varying λ and μ can be used to study the robust character of the consistency of the random variable n, when μ is random variable.

7. Discussion

Here it should be recognized that the basic distribution can be updated in view of prior variations in its parameter(s).Further, on using the experimental data, the prior can be updated in the form of posterior in the Bayesian framework. Still further, the basic distribution can be compounded with the posterior distribution to provide the updated compound or predictive normal distribution. In the process, we get the two basic normal distribution of X in (1) and (4) representing two specific situations. As is evident from the analysis presented in section 4, this predictive normal distribution enables us to study the robust characters of SNTP in respect of α, β and ASN function when prior has been updated with posterior as its distribution. For meeting this objective, we compare the generalized form of OC. curve in (6) with its particular form given in (10). Similarly, the generalized form of ASN in (7) is compared with its particular form in (12) so the last $C.V_\mu(n)$ and $C.V_\lambda(n)$ as given in (19) and (22) respectively are compared to analysis the robust character of the consistency of the random variable n when μ is considered as a random variable with posterior as its distribution. For clarity, we consider an example.

8. An Example

We have considered the SPRT for testing the hypothesis $H_0:\mu=\mu_0=\lambda_0=50$ v/s $H_1:\mu=\mu_1=\lambda_1=47$ when observations are sequential recorded from $N\left(50,\sigma_1^2=64\right)$ and that for μ in (2) will be $N\left(50,\sigma_2^2=36\right)$, further suppose that the process sample information $x_1,x_2,\ldots\ldots x_n$ yields $\bar{x}=53$ for n = 25, Then on using (3) and (4), we have

$$\lambda=\frac{n\bar{x}\sigma_2^2+\theta\sigma_1^2}{\sigma_1^2+n\sigma_2^2}=52.80$$

$$\left[\beta^2=\sigma_1^2+\xi^2\right]$$

$$\beta^2=66.39$$

In other words, the predictive normal distribution of X becomes $N(52.80,\ 66.39)$. Now on using the expression in (6), (7) and (19), a few respective point on $L(\mu)$, $E_\mu(n)$ and $C.V_\mu(n)$ for varying μ have been summarized in Table-1. Now on considering the predictive basic distribution of X in (4), the expression in (10), (12) and (22) the respective points on $L(\lambda)$, $E_\lambda(n)$ and

$C.V_{\lambda}(n)$ have also been listed in Table-1. For still better analysis of the non- robust character of SNTP in respect of α, β and ASN, when posterior prior has been used, the points in Table-1 are used to draw the curves for $L(\mu)/L(\lambda)$, $E_{\mu}(n)/E_{\lambda}(n)$ and $C.V_{\mu}(n)/C.V_{\lambda}(n)$ for varying $\mu = E(\mu) = \lambda$ in figure 1, 2 and 3 respectively.

9. Analysis

According to A. Wald [1947], the basis of selecting a desirable sequential test involves the following steps.

- We first consider admissible sequential test of the same strength (α, β).
- Then from among the admissible test, A test provide minimum $E_{\mu}(n)$, for all values of μ is considered as most desirable.

Table 1: Showing values of $L(\mu)/L(\lambda)$, $E_{\mu}(n)/E_{\lambda}(n)$ and $C.V._{\mu}(n)/C.V._{\lambda}(n)$ for varying $\mu = E(\mu) = \lambda$

μ = λ = Θ	L (μ)	L (λ)	E_{μ} (n)	E_{λ}(n)	$C.V_{\mu}$(n)	$C.V_{\lambda}$(n)
46.5	0.021	0.021	13.497	13.554	101.267	110.940
47	0.049	0.049	16.753	18.073	119.832	120.056
47.25	0.079	0.079	18.701	19.063	131.789	135.315
47.5	0.125	0.125	21.086	21.227	152.380	155.000
47.75	0.187	0.187	23.016	23.620	196.423	182.621
48	0.275	0.275	24.907	25.473	253.045	259.476
48.25	0.384	0.384	26.277	26.277	481.000	483.316
48.75	0.615	0.615	26.050	26.050	477.233	480.000
49	0.725	0.725	24.793	25.473	252.126	259.476
49.25	0.813	0.813	22.941	23.620	184.769	190.512
49.5	0.874	0.874	20.831	21.171	150.885	154.376
49.75	0.92	0.92	18.701	19.018	131.578	135.000
50	0.949	0.949	16.753	16.941	116.167	122.000
50.5	0.978	0.978	13.497	13.526	101.015	102.767

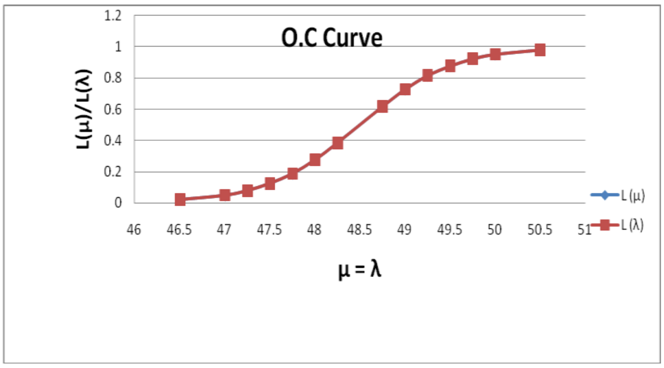

Fig.1:OC Curve for SNTP

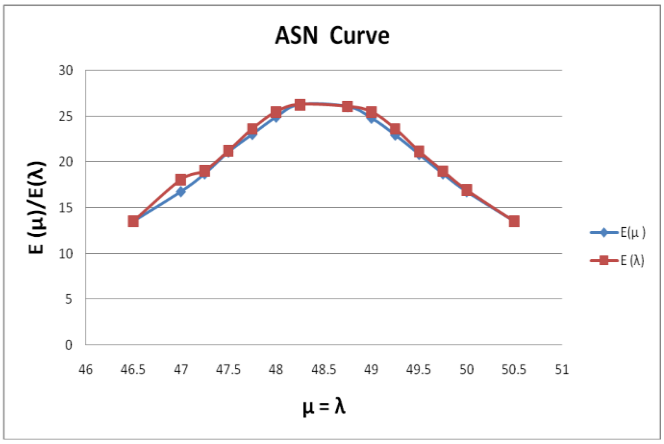

Fig.2: ASN Curve for SNTP

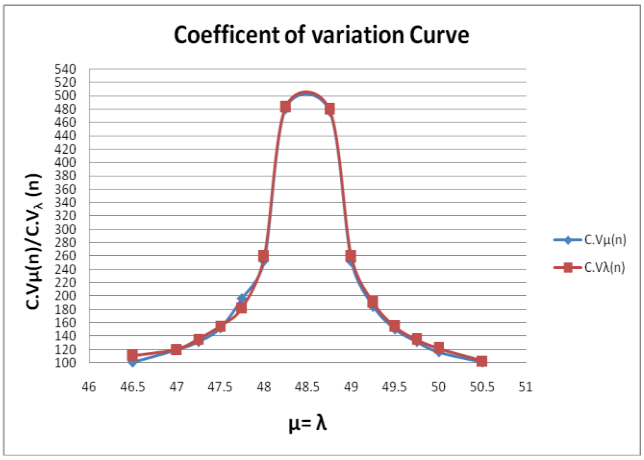

Fig.3: Coefficent of variation curve for SNTP

10. Conclusion

However, the criterion suffers from the following limitations.

(a) In general, no uniformly test exist depending on $E_{\mu}(n)$.

(b) ASN function, $E_{\mu}(n)$ alone in unable to reflect the consistent behaviour [closeness between n and $E_{\mu}(n)$] of n as we updating 'μ' in view of prior variation.

To overcome these limitations, a suitable measure of relative dispersion called $CV_{\mu}(n)$ is used to analyze this consistency (closeness) between n and $E_{\mu}(n)$. Based on this criterion, an admissible sequential test of strength (α, β) with maximum consistency is called a desirable test.

In the light of above developments, we are able to conclude the following important results when posterior has been updated on using experimental data,

- SNTP are non-robust in respect of α, β, $E_{\mu}(n)$ and when μ is considered as a random variable. In the present data setup and from fig. (2), we observe that $E_\lambda(n)$ tends to be uniformly higher than $E_\mu(n)$ for all varying values of μ = E (μ) = λ. However, the random variable n tend to be uniformly less consistent as $C.V._\lambda(n)$ is uniformly higher than $CV_\mu(n)$ for values of μ = E (μ) = λ < 47. On the other hand, the random variable n tend to be uniformly more consistent consistent or robust.

The conclusion is that SNTP are found to be non robust in respect of α, β, ASN and consistent behaviour of n, when μ is considered as a random variable. Therefore, NTP be used cautiously whenever there is a concern about the random variations in μ.

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as $C.V._\lambda(n)$ is uniformly less than $CV_\mu(n)$ for values of μ = E(μ)= λ > 50

- A comparison of the two OC curves, $L(\mu)$ and L(λ) , in fig.(1) reveals that α and β both tend to be equal for all values of μ = E(μ)= λ .
- However the values of μ and λ around the values μ= λ =47 or 50, the SNTP are observed to be equally

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