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A Study on Various Connectivity in Fuzzy Graph

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Abstract— Fuzzy graph theory is an important extension of classical graph theory introduced to model systems involving uncertainty. In fuzzy graphs, vertices and edges are associated with membership values between 0 and 1. Connectivity is one of the most fundamental structural properties of fuzzy graphs. It helps in understanding the relationship between vertices and the strength of connections between them. In this paper, various types of connectivity in fuzzy graphs are studied. Important definitions, properties, and theorem results related to fuzzy connectivity and strong fuzzy connectivity are presented. The structural properties of connected fuzzy graphs are analyzed. These results help in understanding fuzzy networks and their applications in real-world systems.

Keywords *— Fuzzy Graph; Fuzzy Connectivity; Strong Fuzzy Graph; Fuzzy Path; Graph Theory.*

1. Introduction

Graph theory is an important area of mathematics used to study relationships between objects. A graph consists of vertices and edges connecting pairs of vertices. Classical graph theory assumes that relationships are precise and exact. However, many real-world systems involve uncertainty. Fuzzy graph theory was introduced to handle such uncertainty. In fuzzy graphs, vertices and edges have membership values between *0* and *1*. Connectivity is an important concept in fuzzy graph theory. It describes whether vertices are connected through paths. Connectivity helps in understanding structural properties of fuzzy graphs. Fuzzy graph connectivity has applications in communication networks, computer science, engineering, and decision-making systems. This paper presents definitions, properties, and theorem results related to connectivity in fuzzy graph.

2. Preliminaries and Definitions

Definition

Fuzzy Set Let X be a non-empty set. A fuzzy set A in X is defined by a membership function $\mu : X \rightarrow [0,1]$ where $\mu(x)$ represents the degree of membership of element x in A.

Definition

Fuzzy Graph A fuzzy graph G is defined as $G = (V, \sigma, \mu)$ where: $\sigma : V \rightarrow [0,1]$ is the vertex membership function $\mu : V \times V \rightarrow [0,1]$ is the edge membership function such that: $\mu(u, v) \leq min(\sigma(u), \sigma(v))$.

Definition

Fuzzy Path A fuzzy path is a sequence of vertices connected by edges having non-zero membership values.

Definition

Connected Fuzzy Graph A fuzzy graph is connected if there exists a path between every pair of vertices

Definition

Strong Fuzzy Graph A fuzzy graph is called strong if: $\mu(u, v) = min(\sigma(u), \sigma(v))$ for all vertices u and v.

Definition

Complete Fuzzy Graph A fuzzy graph is complete if there exists an edge between every pair of vertices.

3. Connectivity In Fuzzy Graph

Definition

A vertex w in a fuzzy graph is called a cyclic cut vertex if cyclic cut

$(\boldsymbol{G - W})$.

Theorem

For a complete fuzzy graph $\boldsymbol{G, K} \subset (\boldsymbol{G})$

Proof

Given complete fuzzy graph G with vertices

$\boldsymbol{v_1, v_2, \dots \dots}$

Such that

$\boldsymbol{ds(v_{1)} \leq ds(v_{2)} \leq \cdots \dots . ds(v_n)}$

Let$\boldsymbol{v_1}$ be a vertex. $\boldsymbol{ds(v_1) = s(G)}$

Case I: If $\boldsymbol{v_1}$is a cyclic cut vertex.

Here, $\boldsymbol{V = \{v_1\}}$ is a cyclic cut set of $\boldsymbol{G}$.

Therefore,

$\boldsymbol{S \subset (V) = min\,(v_1, v_i)\; for\; i = (2, \dots \dots . n)}$

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$=\mu\ (v_1\ ,v_i\ )$

$= s\ (G)$

Now,

Since $K\ \subset\ (G)$= min {S c (V)},
Where V is a cyclic cutest of G.

We have $K\ \subset\ (G)$

Case II:

If v_1 is not a cyclic cut vertex.

Let F = $\{u_1\ ,u_2\ ,\ \ldots\ldots.\}$, be a cyclic cut set such that

$S\ \subset\ (F)\ =\ K\ \subset\ (G).$

Now, $K\ \subset\ (G) = S\ \subset\ (F)$

$=\sum min\ \{\mu(u_i),\mu(u_j)\}$, for all u_i, u_j, for i, j=1, 2, ……. n

$=\sum min\ \{\ \mu(u_i,u_j)\ \}$

$= ds(v_{1)}$

$= \delta\ s\ (G)\qquad = K(G)$

Hence ProvedConnectivity describes the existence of paths between vertices. Connectivity helps in understanding structural properties. Connectivity determines:

- Reachability
- Structural relationship
- Network reliability
- Graph structure Connectivity is an important property in fuzzy graph theory.

4. Main Theorems

Theorem

Every strong fuzzy graph is connected.

Proof:

In a strong fuzzy graph, edge membership values are equal to the minimum of vertex membership values. This ensures maximum connectivity. Therefore, vertices are connected.

5. Properties of Connected Fuzzy Graph

Connected fuzzy graphs satisfy: Path existence between vertices Structural connectivity Network stability Graph completeness These properties help in analyzing fuzzy graph structure.

6. TYPES OF CONNECTIVITY

Fuzzy graphs have different types of connectivity:

- Strong connectivity
- Weak connectivity
- Complete connectivity

These connectivity types help in structural analysis.

7. Applications of Fuzzy Graph Connectivity

Fuzzy graph connectivity is used in:

- Communication networks
- Artificial intelligence
- Computer networks
- Decision-making systems
- Engineering systems Connectivity helps in analyzing network reliability.

8. Conclusion

In this paper Criterion for connectivity of a fuzzy graph is analyzed. The concept of connectivity and cycle connectivity play an important role in fuzzy graph theory. We have noted down the existence if different types of arcs to categorize Regular and Totally Regular Fuzzy Graphs and Fuzzy trees and Fuzzy cycle and Spanning trees are also discussed.

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