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Analysis of Impulsive Response of Mechanical and Electrical Oscillators by Rohit Transform

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Abstract — The analysis of impulsive responses of mechanical and electrical oscillators is a well-known problem in applied sciences and engineering. In this paper, the impulsive responses of mechanical and electrical oscillators are obtained by the new integral transform 'Rohit transform'. This paper brings up the Rohit transform as a new technological approach for obtaining the impulsive responses of mechanical and electrical oscillators. The results obtained by applying the Rohit transform are the same as obtained with other integral transforms or approaches.

Keywords — *Mechanical and Electrical Oscillators; Impulsive Responses; Rohit Transform.*

1. Introduction

The mechanical and electrical oscillators are generally analyzed by different integral transforms and approaches [1]-[12]. This paper analyses the mechanical and electrical oscillators by new integral transform 'Rohit transform' to obtain their impulsive responses. The Rohit Transform is a new integral transform which has been proposed by the author Rohit Gupta in the recently in the year 2020 and generally, it has been applied to boundary value problems in most of the science and engineering disciplines such as for the analysis of radioactive decay problem [13], for solving the Schrodinger equation [14], for the Analysis of RLC circuits with exponential excitation sources [15], for the Analysis of Basic Series Inverter [16], for the analysis of Electric Network Circuits with Sinusoidal Potential Sources [17], for the analysis of uniform infinite fin [18], Response of RLC network circuit with steady source [19], for the analysis of one-way streamline flow between parallel plates [20], Response of a Permanent Magnet Moving Coil Instrument [21], Heat Conducted Through Fins Of Varying Cross-Sections [22], Response of an Undamped Forced Oscillator [23], Analysis Of Damped Mechanical And Electrical Oscillators [24]. The purpose of paper is to demonstrate the Rohit transform for analyzing the impulsive responses of mechanical and electrical oscillators. The energy of a simple harmonic oscillator persists indefinitely without the reduction of amplitude and remains constant throughout the motion. In practice, the oscillator is acted upon by the frictional forces arising from the viscosity of the medium or from within the system itself which causes the reduction of amplitude and hence the energy of the oscillator with time [1], [10]-[12]. In electrical oscillator, energy is dissipated due to the presence of resistance of the circuit [7]-[9]. However, the presence of small damping force does not have any significant effect on the undamped oscillations of the oscillator. In such a case, the frictional forces acting on the oscillator are directly proportional to the velocity of the oscillator only [1]-[12].

The Rohit transform of g(y), y ≥ 0 is denoted by $G(r)$ and is given by,

$$G(r) = r^3 \int_0^\infty e^{-ry}\, g(y)\, dy,$$

provided the integral is convergent, where r may be a real or complex parameter [14]-[17].

The RT of some elementary functions are

- $R\{y^n\} = \frac{n!}{r^{n-2}}$, $where\ n = 0,1,2,3 \ldots\ldots$
- $R\{e^{by}\} = \frac{r^3}{r-b}$, $r > b$
- $R\{sinby\} = \frac{b\,r^3}{r^2+b^2}$, $r > 0$
- $R\{sinhby\} = \frac{b\,r^3}{r^2-b^2}$, $r > |b|$
- $R\{cosby\} = \frac{r^4}{r^2+b^2}$, $r > 0$
- $R\{coshby\} = \frac{r^4}{r^2-b^2}$, $r > |b|$
- $R\{\delta(y)\} = r^3$, where $\delta(y)$ is Dirac delta function.

1.1 Some Properties Rohit Transform

$$\{\delta(y)\} = r^3 \int_0^\infty e^{-ry}\, \delta(y)\, dy$$

Since the Dirac delta function [1], [12] $\delta(y)$, is defined as $\delta(y) = 0\ when\ y \neq 0\ and\ \int_{-\infty}^{\infty} g(y)\,\delta(y)\,dy = g(0)$, therefore, the above integral can be rewritten as

$R\{\delta(y)\} = r^3 e^{-r(0)}$
$R\{\delta(y)\} = r^3$
Hence $R\{\delta(y)\} = r^3$
Some properties of the Rohit transform on partial derivatives are discussed as below:

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The Rohit Transform (RT) of first derivative of g(y) i.e. RT of $\frac{\partial g(y)}{\partial y}$ w.r.t. y, (y ≥ 0) [14]-[16] is given by

$$R\left\{\frac{\partial g(y)}{\partial y}\right\} = r^3\int_0^\infty e^{-ry}\frac{\partial g(y)}{\partial y}dy$$

Integrating by parts and applying limits, we get

$$R\left\{\frac{\partial g(y)}{\partial y}\right\} = r^3\left\{-g(0) - \int_0^\infty -re^{-ry}g(y)dy\right\}$$

$$R\left\{\frac{\partial g(y)}{\partial y}\right\} = r^3\left\{-g(0) + r\int_0^\infty e^{-ry}g(y)dy\right\}$$

$$R\left\{\frac{\partial g(y)}{\partial y}\right\} = r^3\{-g(0) + rR\{g(y)\}\}$$

$$R\left\{\frac{\partial g(y)}{\partial y}\right\} = rG(r) - r^3g(0)$$

Hence $R\left\{\frac{\partial g(y)}{\partial y}\right\} = rG(r) - r^3g(0)$

Since $R\left\{\frac{\partial g(y)}{\partial y}\right\} = rR\{g(y)\} - r^3g(0)$,

therefore, on replacing g(y) by $\frac{\partial g(y)}{\partial y}$ and $\frac{\partial g(y)}{\partial y}$ by $\frac{\partial^2 g(y)}{\partial y^2}$, we have

$$R\left\{\frac{\partial^2 g(y)}{\partial y^2}\right\} = rR\left\{\frac{\partial g(y)}{\partial y}\right\} - r^3\frac{\partial g(0)}{\partial y}$$

$$R\left\{\frac{\partial^2 g(y)}{\partial y^2}\right\} = r\{rR\{g(y)\} - r^3g(0)\} - r^3\frac{\partial g(0)}{\partial y}$$

$$R\left\{\frac{\partial^2 g(y)}{\partial y^2}\right\} = r^2R\{g(y)\} - r^4g(0) - r^3\frac{\partial g(0)}{\partial y}$$

$$R\left\{\frac{\partial^2 g(y)}{\partial y^2}\right\} = r^2G(r) - r^4g(0) - r^3\frac{\partial g(0)}{\partial y}$$

Hence $R\left\{\frac{\partial^2 g(y)}{\partial y^2}\right\} = r^2G(r) - r^4g(0) - r^3\frac{\partial g(0)}{\partial y}$

Thus the RT of derivatives of g(y) with respect to y (y ≥ 0) are given by

$R\left\{\frac{\partial g(y)}{\partial y}\right\} = rG(r) - r^3g(0)$,

$R\left\{\frac{\partial^2 f(y)}{\partial y^2}\right\} = r^2G(r) - r^4g(0) - r^3g'(0)$,

$R\left\{\frac{\partial^3 f(y)}{\partial y^3}\right\} = r^3G(r) - r^5g(0) - r^4g'(0) - r^3g''(0)$,

And so on.

2. Material and Method

2.1 Mechanical (Damped) Oscillator

The differential equation of damped oscillator subjected to the impulsive force [1], [3], [12] is given by $\ddot{x}(t) + 2a\dot{x}(t) + \omega^2x(t) = g(t)$ …. (1), where $g(t) = \gamma\delta(t)$, $\delta(t)$ is a delta function (impulsive force per unit mass) and γ is its strength, $2a = \frac{r}{m}$ represents the damping constant per unit mass, $\omega = \sqrt{\frac{K}{m}}$ represents the natural frequency of the oscillator. For a lightly damped oscillator, $a < \omega$.

The initial boundary conditions as follows [3]-[5], [25], [26]:

i. If we measure the time from the instant when the oscillator is crossing its mean position, then at t = 0, x (0) = 0.
ii. Also, at the instant t = 0^+ (i.e. just after applying the impulsive force), we assume that the velocity of the oscillator is maximum i.e.$\dot{x}(0^+) = v_0$.

The Rohit transform of (1) provides

$$q^2\bar{x}(q) - q^4x(0) - q^3\dot{x}(0) + 2a\{q\bar{x}(q) - q^3x(0)\} + \omega^2\bar{x}(q) = \gamma q^3 \quad ..(2)$$

Here $\bar{x}(q)$ denotes the Rohit transform of $x(t)$.

Applying boundary conditions $x(0) = 0$ and $\dot{x}(0) = v_0$ and simplifying (2), we get

$$\bar{x}(q) = \frac{(\gamma + v_0)q^3}{q^2 + 2a\,q + \omega^2}$$

Or

$$\bar{x}(q) = \frac{(\gamma + v_0)q^3}{(q+\beta_1)(q+\beta_2)}$$

where $\beta_1 = a + i\sqrt{\omega^2 - a^2}$ *and* $\beta_2 = a - i\sqrt{\omega^2 - a^2}$ such that $\beta_1 - \beta_2 = 2i\sqrt{\omega^2 - a^2}$

.

Or $\bar{x}(q) = \frac{(\gamma+v_0)q^3}{(\beta_2-\beta_1)(q+\beta_1)} - \frac{(\gamma+v_0)q^3}{(\beta_2-\beta_1)(q+\beta_2)}$

Applying inverse Rohit Transform, we get

$$x(t) = (\gamma + v_0)\frac{[e^{-\beta_1 t} - e^{-\beta_2 t}]}{(\beta_2 - \beta_1)} \quad …….. (3)$$

To find γ, applying condition $\dot{x}(0) = v_0$ and solving, we get $x = 0 \ldots\ldots (4)$

Using (4) in (3), we get

$$x(t) = v_0\frac{[e^{-\beta_1 t} - e^{-\beta_2 t}]}{(\beta_2 - \beta_1)}$$

Or

$$x(t) = v_0e^{-at}\frac{[e^{i\sqrt{\omega^2-a^2}t} - e^{-i\sqrt{\omega^2-a^2}t}]}{2i\sqrt{\omega^2-a^2}}$$

Or

$$x(t) = \frac{v_0e^{-at}}{\sqrt{\omega^2-a^2}}\sin\sqrt{\omega^2 - a^2}\,t \quad …. (5)$$

When $r = 0, a = \frac{r}{2m} = 0$, then (5) reduces to

$$x(t) = \frac{v_0}{\omega}\sin\omega t$$

The equation (5) provides the impulsive response of a lightly damped oscillator and reveals that it is independent

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on the strength of the impulsive force. Also, the behavior of oscillator is oscillatory with amplitude of oscillations reduces with time exponentially. When the damping force is zero, the amplitude of oscillations remains constant.

For an overdamped oscillator [5], $a > \omega$, therefore, replacing $\sqrt{\omega^2 - a^2}$ by $i\sqrt{a^2 - \omega^2}$ in (5), the displacement of an overdamped oscillator is given by

$$x(t) = \frac{v_0 e^{-at}}{i\sqrt{a^2-\omega^2}} \sin i\sqrt{a^2 - \omega^2}\, t$$

Or

$$x(t) = \frac{v_0 e^{-at}}{\sqrt{a^2-\omega^2}} \sinh \sqrt{a^2 - \omega^2}\, t \; .... (6)$$

This equation (6) provides the impulsive response of heavily damped oscillator and reveals that the motion of heavily damped oscillator is non-oscillatory.

2.2 Electrical (Damped) Oscillator

The differential equation of electrical oscillator (LRC circuit) subjected to the impulsive Potential [8], [9], [12] is given by

$\ddot{Q}(t) + 2a\dot{Q}(t) + \omega^2 Q(t) = g(t)$ .... (7), where $g(t) = \gamma\,\delta(t)$, where $g(t) = \gamma\delta(t)$, $\delta(t)$ is a delta potential (impulse) and γ is its strength, $\omega = \sqrt{\frac{1}{LC}}$ represents the angular frequency of the electrical oscillator, $2a = \frac{R}{L}$ represents the damping coefficient. $Q(t)$ is the instantaneous charge.

The initial boundary conditions [10], [11] as follows:

(i) At t = 0, Q (0) = 0.

(ii) Also, at the instant $t = 0^+$ (i.e. just after applying the impulsive potential, we assume that the current in the circuit is maximum i.e. $\dot{Q}(0^+) = i_0$.

The Rohit transform of (1) provides

$$q^2\bar{Q}(q) - q^4 Q(0) - q^3\dot{Q}(0) + 2a\{q\bar{Q}(q) - q^3 Q(0)\} + \omega^2\bar{Q}(q) = \gamma\, q^3 ..(8)$$

Here $\bar{Q}(q)$ denotes the Rohit transform of $Q(t)$.

Applying boundary conditions [12] $Q(0) = 0$ and $\dot{Q}(0) = i_0$ and simplifying (8), we get

$$\bar{Q}(q) = \frac{(\gamma + i_0)q^3}{q^2 + 2a\,q + \omega^2}$$

Or

$$\bar{Q}(q) = \frac{(\gamma + i_0)q^3}{(q + \beta_1)\,(q + \beta_2)}$$

where $\beta_1 = a + i\sqrt{\omega^2 - a^2}$ *and* $\beta_2 = a - i\sqrt{\omega^2 - a^2}$ such that $\beta_1 - \beta_2 = 2i\sqrt{\omega^2 - a^2}$

.

Or $\bar{Q}(q) = \frac{(\gamma+i_0)q^3}{(\beta_2-\beta_1)(q+\beta_1)} - \frac{(\gamma+i_0)q^3}{(\beta_2-\beta_1)(q+\beta_2)}$

Applying inverse Rohit Transform, we get

$$Q(t) = (\gamma + i_0)\frac{[e^{-\beta_1 t} - e^{-\beta_2 t}]}{(\beta_2-\beta_1)} \; .... (9)$$

To find γ, applying condition $\dot{Q}(0) = i_0$ and solving, we get $\gamma = 0 \; ... \; ....(10)$

Using (10) in (9), we get

$$Q(t) = i_0\frac{[e^{-\beta_1 t} - e^{-\beta_2 t}]}{(\beta_2-\beta_1)}$$

Or

$$Q(t) = i_0 e^{-at}\frac{[e^{i\sqrt{\omega^2-a^2}\,t} - e^{-i\sqrt{\omega^2-a^2}\,t}]}{2i\sqrt{\omega^2-a^2}}$$

Or

$$Q(t) = \frac{i_0 e^{-at}}{\sqrt{\omega^2-a^2}} \sin\sqrt{\omega^2 - a^2}\, t .... (11)$$

When $R = 0$, $a = \frac{R}{2L} = 0$, then (11) reduces to

$$Q(t) = \frac{i_0}{\omega}\sin\omega t$$

The equation (11) provides the impulsive response of an electrical oscillator and reveals that it is independent on the strength of the impulsive potential. Also, the behavior of oscillator (charge) is oscillatory with the amplitude of oscillations reduces with time exponentially. The decrease in amplitude i.e. damping depends upon resistance R in the circuit. Such a damping is called resistance damping [2] [9] [12]. If R = 0, the amplitude would remain constant. Hence in LRC circuit, the resistance is the only dissipative element.

3. Conclusion

In this paper, we have successfully applied the Rohit Transform to obtain the impulsive responses of damped mechanical and electrical oscillators and exemplified it for successfully analyzing the theory of damped mechanical and electrical oscillators. A new and different method is exploited for obtaining the impulsive responses of damped mechanical and electrical oscillators.

References

[1] Rahul Gupta, Rohit Gupta, Dinesh Verma, Application of Convolution Method to the Impulsive Response of A Lightly Damped Harmonic Oscillator, International Journal of Scientific Research in Physics and Applied Sciences ,Vol.7, Issue.3, pp.173-175, June (2019).

[2] Fundamentals of Physics by Dr. Robert Resnick and David Halliday, Publisher: John Wiley and sons, 10th edition.

[3] Mathematical physics by E. Butkov, Addison-Wesley Pub Co, 1968, Chapter 7.

[4] Murray R. Spiegel, Theory and Problems of Laplace Transforms, Schaum's outline series, McGraw – Hill.

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[5] Rohit Gupta, Rahul Gupta, Sonica Rajput, Analysis of Damped Harmonic Oscillator by Matrix Method, International Journal of Research and Analytical Reviews (IJRAR), Volume 5, Issue 4, October 2018, pp. 479-484.

[6] Dinesh Verma, Yuvraj Singh, Rohit Gupta, Response of Electrical Networks with Delta Potential via Mohand Transform, International Research Journal of Innovations Engineering and Technology, volume 2, issue 2, February 2020, pp. 41-43.

[7] Rohit Gupta, Loveneesh Talwar, Rahul Gupta, Analysis of R-Ł- C network circuit with steady voltage source, and with steady current source via convolution method, International journal of scientific & technology research, volume 8 Issue 11, November 2019, pp. 803-807.

[8] Dr. J. S. Chitode and Dr. R.M. Jalnekar, Network Analysis and Synthesis, Publisher: Technical Publications, 2007.

[9] Rohit Gupta, Rahul Gupta, Matrix method for deriving the response of a series Ł- C- R network connected to an excitation voltage source of constant potential, Pramana Research Journal, Volume 8, Issue 10, 2018, pp. 120-128.

[10] Rohit Gupta, Rahul Gupta, Sonica Rajput, Convolution Method for the Complete Response of a Series Ł-R Network Connected to an Excitation Source of Sinusoidal Potential, International Journal of Research in Electronics And Computer Engineering, Vol. 7, issue 1, January- March 2019, pp. 658-661.

[11] Rohit Gupta, Rahul Gupta, Sonica Rajput, Response of a parallel Ł- C- $\mathcal{R}$ network connected to an excitation source providing a constant current by matrix method, International Journal for Research in Engineering Application & Management, Vol. 4, Issue 7, Oct. 2018, pp. 212-217.

[12] Rohit Gupta and Loveneesh Talwar, "Elzaki Transform Means To Design A Protective RC Snubber Circuit," International Journal of Scientific and Technical Advancements, Volume 6, Issue 3, 2020, pp. 45-48.

[13] Rohit Gupta, "On Novel Integral Transform: Rohit Transform and Its Application to Boundary Value Problems", ASIO Journal of Chemistry, Physics, Mathematics & Applied Sciences, Volume 4, Issue 1, 2020, 08-13.

[14] Rohit Gupta, Rahul Gupta, Dinesh Verma, Solving Schrodinger equation for a quantum mechanical particle by a new integral transform: Rohit Transform, "ASIO Journal of Chemistry, Physics, Mathematics & Applied Sciences , 4(1), 2020, pp. 32-36.

[15] Rohit Gupta, Rahul Gupta, Analysis of RLC circuits with exponential excitation sources by a new integral transform: Rohit Transform, ASIO Journal of Engineering & Technological Perspective Research, 5(1), 2020, pp.22-24.

[16] Anamika, Rohit Gupta, Analysis Of Basic Series Inverter Via The Application Of Rohit Transform, International Journal of Advance Research and Innovative Ideas in Education, Vol-6, Issue-6, 2020, pp. 868-873.

[17] Loveneesh Talwar, Rohit Gupta, Analysis of Electric Network Circuits with Sinusoidal Potential Sources via Rohit Transform, International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering, 9(11), 2020, pp. 3020-3023.

[18] Neeraj Pandita, Rohit Gupta, Analysis Of Uniform Infinite Fin via Means Of Rohit Transform, , International Journal of Advance Research and Innovative Ideas in Education, 6(6), 2020, pp. 1033-1036.

[19] Neeraj Pandita, Rohit Gupta, Analysis Of Uniform Infinite Fin via Means Of Rohit Transform, , International Journal of Advance Research and Innovative Ideas in Education, 6(6), 2020, pp. 1033-1036.

[20] Gupta R., Gupta R., Verma D. 2022. Response of RLC network circuit with steady source via Rohit transform. International Journal of Engineering, Science and Technology, Vol. 14, No. 1, pp. 21-27. doi: 10.4314/ijest.v14i1.3

[21] Gupta R., Singh I., 2022. Analysis of one-way streamline flow between parallel plates via Rohit integral transform, International Journal of Trendy Research in Engineering and Technology, Vol. 6, No. 5, pp. 29-32.

[22] Rohit Gupta, Rahul Gupta, Anamika, Response of a Permanent Magnet Moving Coil Instrument via the Application of Rohit Transform, Engineering and Scientific International Journal (ESIJ), 8(2), 2021 ,pp. 42-46. doi: 10.30726/esij/v8.i2.2021.82010

[23] Neeraj Pandita and Rohit Gupta, Heat Conducted Through Fins Of Varying Cross-Sect ions Via Rohit Transform, EPRA Journal Of Research And Development, 5(12), 2020, pp. 222-226.

[24] Rohit Gupta, Inderdeep Singh, Ankush Sharma, Response of an Undamped Forced Oscillator via Rohit Transform, International Journal of Emerging Trends in Engineering Research, Volume 10. No.8, 2022, pp. 396-400. doi: 10.30534/ijeter/2022/031082022.

[25] Rohit Gupta Rahul Gupta, Analysis Of Damped Mechanical And Electrical Oscillators By Rohit transform, ASIO Journal Of Chemistry, Physics, Mathematics & Applied Sciences, 4(1), 2020, pp. 45-47.

[26] Rahul Gupta and Rohit Gupta, Impulsive Responses of Damped Mechanical and Electrical Oscillators, International Journal of Scientific and Technical Advancements, 6(3), 2020, pp. 41-44.

[27] Rohit Gupta, Rahul Gupta, Sonica Rajput, Analysis of Damped Harmonic Oscillator by Matrix Method, International Journal of Research and Analytical Reviews, 5(4), 2018, pp. 479-484.