Engineering and Scientific International Journal (ESIJ)
Volume 9, Issue 2, April – June 2022
ISSN 2394-7187(Online)
ISSN 2394 - 7179 (Print)

Analysis of LCR Network Circuits with Exponential Sources by New Integral Transform Gupta Transform

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Abstract — In science and engineering, the analysis of Inductor, Capacitor and Resistor (LCR) network circuits is a principal course and is generally done by applying calculus or integral transforms. In this paper, we will analyze LCR network circuits with exponential excitation sources by applying the new integral transform 'Gupta Transform' and obtain their responses in the form of current or voltage. This paper presents the application of the 'Gupta Transform' and proves its applicability for analyzing the LCR network circuits with exponential excitation sources and concludes that 'Gupta Transform' like other methods or approaches is also an effective and simple tool.

***Keywords* —** *Gupta Transform; Network Circuits; Exponential Source; Response.*

1. Introduction

The Inductor, Capacitor and Resistor (LCR) network circuits are generally analyzed by applying calculus [1-2] or different integral transforms [4-10] or approaches like convolution approach [11-13] or matrix method approach [14-15], or other approaches [16-17]. In this paper, a new integral transform 'Gupta Transform' is presented to analyze the LCR network circuits with exponential excitation sources. The 'Gupta Transform' has been applied in solving boundary value problems in science and engineering [18-21]. This paper presents the application and applicability of 'Gupta Transform' for analyzing the network circuits with exponential excitation sources and concludes that 'Gupta Transform' like other methods or approaches is an effective and simple tool for analyzing the LCR network circuits with exponential sources.

Let g(y) be a continuous function on any interval for y ≥ 0. The Gupta Transform of g(y) is defined as, $\dot{R}\{g(y)\} = \frac{1}{q^3}\int_0^\infty e^{-qy}\, g(y)dy = G(q)$, provided that the integral is convergent, where q may be a real or complex parameter and $\dot{R}$ is the Gupta Transform operator.

1.1 Gupta Transform of Elementary Functions

The Gupta Transform of some elementary functions [19-20] are,

- $\dot{R}\{y^n\} = \frac{n!}{q^{n+4}}$, where $n = 0,1,2,3 \ldots\ldots\ldots$
- $\dot{R}\{e^{by}\} = \frac{1}{q^3(q-b)}$, $r > b$
- $\dot{R}\{sinby\} = \frac{b}{q^3(q^2+b^2)}$, $r > 0$
- $\dot{R}\{sinhby\} = \frac{b}{q^3(q^2-b^2)}$, $r > |b|$
- $\dot{R}\{cosby\} = \frac{1}{q^2(q^2+b^2)}$, $r > 0$
- $\dot{R}\{coshby\} = \frac{1}{q^2(q^2-b^2)}$, $r > |b|$
- $\dot{R}\{\delta(y-b)\} = \frac{1}{q^4}\, e^{-bq}$

1.2 Gupta Transform of Derivatives

Let g(y) is continuous function and is piecewise continuous on any interval, then the Gupta Transform [18-21] of first derivative of g(y) i.e. $\dot{R}\{g'(y)\}$ is given by

$$\dot{R}\{g'(y)\} = \frac{1}{q^3}\int_0^\infty e^{-qy}\, g'(y)\,dy$$

Integrating by parts and applying limits, we get

$$\dot{R}\{g'(y)\} = \frac{1}{q^3}\{-g(0) - \int_0^\infty -qe^{-qy}\, g(y)\,dy\}$$

$$= \frac{1}{q^3}\{-g(0) + q\int_0^\infty e^{-qy}\, g(y)dy\}$$

$$= qG(q) - \frac{1}{q^3}g(0)$$

Hence, $\dot{R}\{g'(y)\} = qG(q) - \frac{1}{q^3}g(0)$.

Since,

$\dot{R}\{g'(y)\} = q\dot{R}\{g(y)\} - \frac{1}{q^3}g(0)$, Therefore, on replacing g(y) by $g'(y)$ and $g'(y)$ by $g''(y)$, we have

$$\dot{R}\{g''(y)\} = q\dot{R}\{g'(y)\} - \frac{1}{q^3}g'(0)$$

$$= q\left\{q\dot{R}\{g(y)\} - \frac{1}{q^3}g(0)\right\} - \frac{1}{q^3}g'(0)$$

$$= q^2\dot{R}\{g(y)\} - \frac{1}{q^2}g(0) - \frac{1}{q^3}g'(0)$$

$$= q^2G(q) - \frac{1}{q^2}g(0) - \frac{1}{q^3}g'(0)$$

DOI: 10.30726/esij/v9.i2.2022.92002

ISSN 2394-7187(Online)
ISSN 2394 - 7179 (Print)

Hence ,
$\dot{R}\{g''(y)\} = q^2 G(q) - \frac{1}{q^2} g(0) - \frac{1}{q^3} g'(0)$ and so on.

2. Materials and Methods

2.1 Series LCR Circuit with Exponential Potential Source

The differential equation for a series LCR circuit with exponential potential source [8, 9] is given by

$$I(t)R + L\dot{I}(t) + \frac{Q(t)}{C} = ve^{-ut} \quad \text{.... (1)}$$

Differentiating (1) w.r.t. t and simplifying we get,

$$\ddot{I}(t) + \frac{R}{L}\dot{I}(t) + \frac{1}{LC} I(t) = \frac{-vu}{L} e^{-ut} \quad \text{.... (2)}$$

Here, $I(t)$ is the instantaneous current in the circuit.

The initial conditions are
(i) $I(t = 0) = 0$ ..... (3)
(ii) Since $I(0) = 0$, therefore, equation (1) gives $\dot{I}(t = 0) = \frac{v}{L}$ ...... (4)

Taking the Gupta transform of equation (2), we get

$$q^2\bar{I}(q) - \frac{1}{q^2} I(0) - \frac{1}{q^3}\dot{I}(0) + \frac{R}{L}\left\{q\bar{I}(q) - \frac{1}{q^3} I(0)\right\} + \frac{1}{LC}\bar{I}(q) = \frac{-vu}{L(q+u)}\frac{1}{q^3} \quad \text{... (5)}$$

Applying conditions: $I(0) = 0$ and $\dot{I}(0) = \frac{v}{L}$ and simplifying (5), we get

$$\bar{I}(q) = \frac{v}{L}\{\frac{1}{q^3}\frac{-u}{(q+u)(-u+\beta_1)(-u+\beta_2)} + \frac{1}{q^3}\frac{-\beta_1}{(-\beta_1+u)(q+\beta_1)(-\beta_1+\beta_2)} + \frac{1}{q^3}\frac{-\beta_2}{(-\beta_2+u)(-\beta_2+\beta_1)(q+\beta_2)}\}$$

$where\ \omega' = \sqrt{\delta^2 - \omega^2}, \delta + \omega' = \beta_1\ and\ \delta - \omega' = \beta_{2,}\ \ \beta_1 - \beta_2 = 2\omega'$

.
Applying inverse Gupta Transform, we get

$$I(t) = \frac{v}{L}\{\frac{-ue^{-ut}}{(-u+\beta_1)(-u+\beta_2)} + \frac{-\beta_1 e^{-\beta_1 t}}{(-\beta_1+u)(-\beta_1+\beta_2)} + \frac{-\beta_2 e^{-\beta_2 t}}{(-\beta_2+u)(-\beta_2+\beta_1)}\}$$

or

$$I(t) = \frac{v}{L}\{\frac{-\beta_1 e^{-\beta_1 t}}{(-\beta_1+u)(-\beta_1+\beta_2)} - \frac{ue^{-ut}}{(-u+\beta_1)(-u+\beta_2)} + \frac{-\beta_2 e^{-\beta_2 t}}{(-\beta_2+u)(-\beta_2+\beta_1)}\}$$

or

$$I(t) = \frac{v}{L}\{\frac{[\delta - \omega']e^{-\delta t}e^{\omega' t}}{2\omega'[\delta - \omega' - u]} - \frac{ue^{-ut}}{[\delta + \omega' - u][\delta - \omega' - u]} - \frac{[\delta + \omega']e^{-\delta t}e^{-\omega' t}}{2\omega'[\delta + \omega' - u]}\} \quad \text{... (6)}$$

This equation (5) gives the response (current) of a series LCR circuit with an exponential potential source at any instant.
When t increases indefinitely, $e^{-\delta t}$ tends to zero, so

$$I(t) = \frac{v}{L}\frac{-ue^{-ut}}{[\delta + \omega' - u][\delta - \omega' - u]}$$

or

$$I(t) = \frac{v}{L}\frac{ue^{-ut}}{[u^2 - \omega^2 - 2\omega']}$$

2.2 Parallel LCR Circuit with an Exponential Current Source

The differential equation for a parallel LCR circuit with an exponential current source [8, 9] is given by

$$\frac{V(t)}{R} + \frac{1}{L}\int V(t)dt + C\dot{V}(t) = I_0 e^{-ut} \quad \text{..... (7)}$$

Differentiate (1) w.r.t. t and simplifying, we get,

$$\ddot{V}(t) + \frac{1}{RC}\dot{V}(t) + \frac{1}{LC}V(t) = \frac{-I_0 u}{C} e^{-ut} \quad \text{.... (8)}$$

The initial conditions are:
1. $V(t = 0) = 0$.

2. Since $V(0) = 0$, therefore, (1) gives $\dot{V}(0) = \frac{I_0}{C}$.

Taking Gupta Transform of (6), we get

$$q^2\bar{V}(q) - \frac{1}{q^2} V(0) - \frac{1}{q^3}\dot{V}(0) + \frac{1}{RC}\left\{q\bar{V}(q) - \frac{1}{q^3} V(0)\right\} + \frac{1}{LC}\bar{V}(q) = \frac{-I_0 u}{C(q+u)}\frac{1}{q^3} \quad \text{....... (9)}$$

Applying conditions: $V(0) = 0$ and $\dot{V}(0) = \frac{I_0}{C}$ and simplifying (3), we get

$$\bar{V}(q) = \frac{I_0}{C}\{\frac{1}{q^3}\frac{-u}{(q+u)(-u+a_1)(-u+a_2)} + \frac{1}{q^3}\frac{-a_1}{(-a_1+u)(q+a_1)(-a_1+a_2)} + \frac{1}{q^3}\frac{-a_2}{(-a_2+u)(-a_2+a_1)(q+a_2)}\}$$

$where\ a + \omega' = a_1\ and\ a - \omega' = a_2, \omega' = \sqrt{a^2 - \omega^2}\ , a_1 - a_2 = 2\omega'$

.
Applying inverse Gupta Transform, we get

DOI: **10.30726/esij/v9.i2.2022.92002**

$$V(t) = \frac{I_0}{C}\left\{\frac{-ue^{-ut}}{(-u+\beta_1)(-u+\beta_2)} + \frac{-\beta_1 e^{-\beta_1 t}}{(-\beta_1+u)(-\beta_1+\beta_2)} + \frac{-\beta_2 e^{-\beta_2 t}}{(-\beta_2+u)(-\beta_2+\beta_1)}\right\}$$

or

$$V(t) = \frac{I_0}{C}\left\{\frac{-\beta_1 e^{-\beta_1 t}}{(-\beta_1+u)(-\beta_1+\beta_2)} - \frac{ue^{-ut}}{(-u+\beta_1)(-u+\beta_2)} + \frac{-\beta_2 e^{-\beta_2 t}}{(-\beta_2+u)(-\beta_2+\beta_1)}\right\}$$

or

$$V(t) = \frac{I_0}{C}\left\{\frac{[\delta-\omega']e^{-\delta t}e^{\omega' t}}{2\omega'[\delta-\omega'-u]} - \frac{ue^{-ut}}{[\delta+\omega'-u][+\delta-\omega'-u]} - \frac{[\delta+\omega']e^{-\delta t}e^{-\omega' t}}{2\omega'[\delta+\omega'-u]}\right\} \dots (8)$$

This equation (8) gives the response (potential) of a parallel LCR circuit with an exponential current source at any instant.

When t increases indefinitely, $e^{-\delta t}$ tends to zero, so

$$V(t) = \frac{I_0}{C}\frac{-ue^{-ut}}{[\delta+\omega'-u][\delta-\omega'-u]}$$

or

$$V(t) = \frac{I_0}{C}\frac{ue^{-ut}}{[u^2-\omega^2-2\,\omega']}$$

3. Result and Conclusion

In this paper, we have successfully obtained the response of network circuits connected to exponential excitation source. This paper exemplified the application of Gupta Transform for obtaining the response of network circuits connected to exponential excitation source. This paper brought up the Gupta Transform as a simple and effective technique for analyzing the network circuits connected to exponential excitation source.

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DOI: **10.30726/esij/v9.i2.2022.92002**