

Three New Models for Ranking of Candidates In the Preferential Voting Systems

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Abstract — Election is the main challenge to the political and social science. In the meantime, in the literature, several methods to decide the winner of elections have been proposed; theoretically there is no reason to be limited to these models. Hence, in this paper, we assume three new approaches (1. election result prediction by pre-election preference information using Markov chain model [to identify the efficient electoral strategy for each candidate]. 2. Improved Borda's function method using the weights of decision makers [or voters]. And 3. A new interval TOPSIS-based approach applying ordinal set of preferences [so, data is ordinal form that first convert to interval value and then inject them into the conventional interval TOPSIS model]) for ranking candidates in voting systems. Ultimately, three numerical examples in social choice context are given to depict the feasibility and practicability of the proposed methods. In sum, this paper suggests a mind line for decreasing the wrong choice winner risks correlated with voting systems.

Keywords — *Voting Systems; Markov Chain Model; Borda's Function; TOPSIS with Interval Data; Ordinal Preference; Ranking of Candidates Problems*

1. Introduction

According to Alam, Mezbahuddin, and Shoma (2015), in the earth, election is very much liking word. When a group of people with individual preferences has to decide which alternative to choose from a given set of alternatives, an election is often carried out (Polykovskiy, Berghammer, & Neumann, 2016). Therefore, obtaining a group ranking or a winning candidate from individual's preferences on a set of alternatives is an important group decision problem with social choice and voting system implications (Aghayi & Tavana, 2019).

In social choice theory, and more particularly in voting theory, a society needs to choose a candidate from a set of candidates (Bouyssou, Marchant, Pirlot, Tsoukias, & Vincke, 2006). Further, social choice theory is a field of scientific inquiry that studies the integration of individual preferences during a collective choice (Brandt, Couitzer, Endriss, Lang, & Procaccia, 2016). Meanwhile, a voting system uses the information provided by the voters in order to determine the elected candidate or, more generally, the decision made by the group (Bouyssou, Marchant, & Perny, 2009).

On the other side, according to Kou and Sobel (2004), electoral outcomes in democratic countries have far-reaching domestic and (sometimes) international impact. Individuals, corporate actors, and governments who anticipate being affected by the outcome of a future election incorporate their expectations (forecasts) into current elections and policies. Outcome prediction of political events is an integral part of the practice and study

of politics. In the social sciences, pure prediction models are used only for a limited number of problems, one of which is elections outcome (Stoltenberg, 2013).

According to Alam, Mezbahuddin, and Shoma (2015), election prediction is very significant for the candidates and the society. Historically, from the 1970s onwards, wide ranges of forecasting techniques have been developed in the literature on electoral forecasting (Walther, 2015). Beck and Dassonneville (2015) believe that, scientific work on national election forecasting has become most developed for the United States case, where three dominant approaches can be identified: structuralists, aggregators, and synthesizers. For European cases, election forecasting models remain almost exclusively structuralists. These methods can be distinguished in terms of their application in theory, data, and time. The structuralists suggest a theoretical model of the election outcome. In contrast, the aggregators, aggregate vote intentions in opinion polls. Taking a different approach, the synthesizers borrow from both the structuralists and the aggregators. In addition, Payne (2001) believes that, three forecasting environments can be identified:

- predicting the final result before the election takes place (the 'pre-forecast'),
- immediately after the polling stations close (the 'prior forecast'),
- During election night itself using the subset of actual results declared (the 'results-based forecast').

The concern here is with the first type of forecasting context (in other words, 'pre-forecast'), by using Markov chain model (as discussed latter in this paper). On the other side, in one hand, according to Ebrahimnejad (2012), in

ranked voting system, each voter selects a subset of candidates and ranked them from most to least preferred. Among these systems, popular procedures to obtain a total ranking or a winning candidate are scoring rules, which fixed score are assigned to the different places. In this way, the score obtained by each candidate is the weighted sum of the points received in places different. The Plurality rule and Borda rule are two well-known examples of the scoring rule. In Plurality rule, the winner candidate receives more votes in the first place. In Borda rule, the weight assigned to the first place equals to number of candidates and to the second place is one less than the first place and so on (or more frequently, $n-1, n-2, \dots, 0$). Ebrahimnejad and Nasseri (2012) believe that, the principal drawback of such scoring rules is that they assume the votes of all voters have equal importance and there is no preference among them. It is the aim of this paper. On the other hand, according Hwang and Lin (1987), it is a Multi Criteria Decision Making process whenever a voter casts a vote to select a candidate or alternative policy. Furthermore, Bouyssou, Marchant, and Perny (2009) believe that, the many results obtained in social choice theory are valuable for Multi Criteria Decision Aiding. There are indeed links between these two domains: it is easy to go from one to the other by replacing the word 'action', 'criterion', 'partial preference', and 'overall preference', by 'candidate', 'voter', 'individual preference' and 'collective preference'. This is the problem we wish to address here. In continuation, a brief discussion of Markov chain model, Borda rule, and Multiple Criteria Decision Making are provided in this section.

According to Talemi, Jahanbani, and Heidarkhani (2013), management is defined decision in a simple form and the most important factor for decision making is forecasting future. In this era, the organizations having high complexity and much information can help to management in a logical and accurate decisions. So, it is easier for managers to use different aspects of Operations Research to deal with complex issues. The Markov chain is one of these models used in Operations Research with the possibility that managers can use it in organizational decision making. Successful decision is an image of the future that this will not be achieved only from the prediction, based on scientific principles. Markov processes is a chain of random events that can be predicted next period by having information of current location and in fact, Markov chain is a tool that employed for forecasting of situation organization in future periods. In other words, it is a random process where all information about the future is contained in the present state. In addition, the main components in developing the Markov chain model are state transition matrix and probability; both will summarize all the essential parameters and dynamic changes (Zakaria, Othman, Sokkalingam, Daud, Abdullah, & Kadir, 2019).

The Borda method is based on a majority rule binary relation (Hwang & Yoon, 1981). So, rank of each pair in different ranking way is compared with each other (Azadfallah, 2016). Further, it is based on the concept of voting and it compares each pair of alternatives separately and forms an $N \times N$ matrix. For each pair of alternatives A_j and A_i , the number of votes is defined as the number of "supporting" methods in which A_j is more preferable than A_i . Then an $N \times N$ matrix is generated such that $X_{ij}=1$, if A_j receives more votes than A_i , $X_{ij}=0$, otherwise. S_j indicates the number of "wins" that A_j has received against other alternatives and it is calculated by summing the X_{ij} in each row of the matrix. Hence, the alternative with the highest S_j is considered the most preferable (Azadfallah, 2019).

On the other side, decision making is the process of identifying and selecting from among possible solution to a problem according to the demands of the situation (Al-Tarawneh, 2012). Multiple Criteria Decision Making (MCDM) deals with decision situations where the decision maker has several-usually conflicting-objectives (Habenicht, Scheubrein, & Scheubrein, 2009). Generally, MCDM can be described as follows: the screening, prioritizing, ranking or selecting the alternatives based on human judgment from among a finite set of decision alternatives in terms of multiple usually conflicting criteria (Roszkowska, 2013), and is one of the most widely use decision methodologies in the sciences, business, and engineering worlds (Azadfallah, 2019). The main steps in MCDM are the following (Opricovic & Tzeng, 2004):

- Selection of the related criteria/attributes,
- generating alternatives,
- Evaluate alternatives in terms of attributes,
- Selection of the appropriate MCDM models,
- Accept one alternative as "optimal" (preferred),
- If the final solution is not accepted, gather new information and go to the next iteration of multi-criteria optimization.

According to Roszkowska (2011), solving of each multi-criteria problem (individual or group decision) begins with building a decision-making matrix (or matrices). In each matrix, values of the criteria for alternatives may be exact, intervals numbers, fuzzy numbers or qualitative labels. Let us denote by $D = \{1, 2, \dots, k\}$ a set of decision makers or experts. The multi-criteria problem can be expressed in k -matrix format in the following way:

$$\begin{array}{c}
 A_1 \\
 A_2 \\
 \dots \\
 A_m
 \end{array}
 \begin{array}{cccc}
 C_1 & C_2 & \dots & C_n \\
 \left| \begin{array}{cccc}
 x_{11}^k & x_{12}^k & \dots & x_{1n}^k \\
 x_{21}^k & x_{22}^k & \dots & x_{2n}^k \\
 \cdot & \cdot & \cdot & \cdot \\
 x_{m1}^k & x_{m2}^k & \dots & x_{mn}^k
 \end{array} \right.
 \end{array}$$

Where:

- $A_1, A_2 \dots A_m$ are possible alternatives that decision makers have to choose from,
- $C_1, C_2 \dots C_n$ are the criteria for which the alternative performance is measured,
- X_{ij}^k is the k -decision maker rating of alternative A_i with respect to the criterion C_j (x_{ij}^k is numerical, interval data or fuzzy number).

In this way for m alternatives and n criteria, we have matrix $x^k = (x_{ij}^k)$ where x_{ij}^k is value of i -alternative with respect to j -criterion for k -decision maker, $j=1, 2 \dots n, k=1, 2 \dots k$.

The relative importance of each criterion is given by a set of weights, which are normalized to sum to one. Let us denote by $w_k = [w_1^k, w_2^k \dots w_n^k]$ a weight vector for k -decision maker, where $w_j^k \in \mathbb{R}$ is the k -decision maker weight of criterion C_j and $w_1^k + w_2^k + \dots + w_n^k = 1$.

In the case of one decision maker, we write x_{ij}, w_j, x , respectively.

According to Jahanshahloo, Lotfi, and Davoodi (2009), there are several methods for solving MCDM problems. One of the most common ways of MCDM is TOPSIS (Dizaji & Khanmohammadi, 2016). The merits of TOPSIS are as follows (Fallahpour, 2016):

- it is a simple technique,
- it takes any kind of attribute,
- the calculation processes are easy,
- It is reasonable and logical.

The TOPSIS procedure begins with the formation of the decision matrix and represents the satisfaction value of each criterion with an alternative. Next, the matrix is normalized with a desired normalizing formula, and the values are multiplied by the criteria weights. Consequently, the positive-ideal and negative-ideal solutions are calculated, and the distance of each alternative to the solutions is identified with a Euclidean distance measure. Finally, the alternatives are ranked in terms of their relative closeness to the ideal solution. The TOPSIS model is useful for decision makers (DMs) to structure the problems to be solved, conduct analyses, comparisons and ranking of the alternatives (Roszkowska, 2011). In some cases, it is not possible to gather exact data, so decision making based on these data loses its efficiency. Hence, Jahanshahloo, Lotfi, and Izadikhah (2006) extended TOPSIS for decision making problems with interval data. The present paper addresses this problem. In continuation, a few points are worth mentioning with respect to the proposed methods.

At first, till now, forecasting models are mainly used to predict electoral results from the past election results.

But in this paper to solve this problem, we have used pre-election preferences (notice, in this paper, we will use the terms pre-election and survey or demanding for other's view to emphasize the same concept) information (a few weeks/ months before the election) to predict election results. So, by utilizing Markov model we will discover efficient electoral strategy for each potential candidate. In addition, in the voting system literature, it is proposed that the votes of all voters have equal significance and there is no preference among them. On the other hand, in some cases, there shall be a priority among voters. So, in the proposed method (the improved Borda's method, using the weights of decision maker) it is assumed that the voters are not equally paramount. As the third point, voting systems is thought to be a MCDM problem. Therefore, in this paper, to solve this problem a new MCDM approach is proposed. Furthermore, we presume that the rating of each alternative and the weight of each criterion are expressed in ordinal forms (in consistency with voting systems). Hence, we first change an ordinal MCDM problem into an interval one via Wang et al. (2005) method and then solve the non-ordinal MCDM problem using the interval version of the TOPSIS method.

To sum up, the contribution of this paper is to take benefit of a numerical example to show the process of the proposed method in voting systems context. The paper is organized as follows. In section 2, the literature is discussed. In section 3 and section 4, the research gap, and the proposed approach is discussed, respectively. Numerical example is provided in section 5. The findings and the conclusion of the paper is presented in section 6 and section 7.

2. Literature Review

In this section, we review previous related researches, and to integrate the survey in various aspects, we divided it into three parts: i) forecasting model, ii) Borda's rule, and iii) MCDM methods in voting systems context.

2.1 Forecasting Model & Voting Systems

According to Walther (2015), predicting the outcome of election is a relatively recent and increasingly popular part of political science research. Nevertheless, Payne (2001) reviewed the various statistical methods used by the BBC to forecast different types of election in the UK (United Kingdom) in the last thirty. Kou and Sobel (2004) developed a model for using both election markets and public opinion polls to forecast electoral outcomes, giving conditions under which all method performs ideally. Nagadevara (2005) employed predictive models (based on the classification Trees and Neural Networks) for election result in India. Nicholson (2005) aimed to identify

paradoxes and to reduce the number of paradoxes in voting. So, when paradoxes arise Markov chains may be created to choose a winner. Hummel and Rothschild (2013 and 2014) developed new fundamental models for forecasting presidential, senatorial, and gubernatorial elections at the state level using basic data from several categories such as previous election outcomes, incumbency, presidential approval ratings, ideological indicators, economic indicators, and biographical information about the candidates. Stoltenberg (2013) introduced a Bayesian-based forecasting model that is more suitable for multiparty systems. Alam, Mezbahuddin, and Shoma (2015) predicted the election results by using hidden Markov model. Beck and Dassonneville (2015) forecasted elections in Europe with synthetic model. Macdonald and Mao (2015) forecasted the 2015 general election with internet big data. Walther (2015) tested whether it is possible the predict elections also in difficult parliamentary systems where a wide range of parties are competing for power, and if this can be done with reasonable lead-time. Kassraie, Modirshanechi, and Aghajan (2017) predicted election vote share using a sentiment-based fusion of twitter data wit Google trends and online polls. Zolghadr, Niaki, and Niaki (2018) modeled and forecasted US presidential election using learning algorithm. Moreover, Colladon (2020) used the semantic brand score (a calculator of brand importance in big textual data) to forecast elections result based on online news.

2.2 Borda's Rule & Voting Systems

According to Egecioglu and Giritligil (2011), the Borda rule is one of the most studied voting procedures in the social choice theory literature. For instance, Debord (1992) presented an extension of Borda's choice function to k-choice function. Breton and Truchon (1997) addressed the difference between the Borda rule and any given social choice function. Lapresta and Panero (2002) considered a fuzzy variant of the Borda count taking into account agents' intensities of preference. Saari (2006) studied which is better; the Condorcet or Borda winner. Nurmi (2007) assessed Borda's rule and its modifications. Lapresta, Panero, and Meneses (2008) used linguistic labels as input in the Borda count. Egecioglu and Giritligil (2011) studied the likelihood of choosing the Borda-winner with partial preference rankings of the electorate. Xia (2011) surveyed developments in generalized scoring rules (further showing that they provide a fruitful framework to obtain general results) and also reconcile the Borda approach and Condorcet approach via a new social choice axiom. Koffi (2015) introduced the generalized partial Borda count voting system, and explore which properties of partial Borda are still satisfied in this general setting. Bag, Azad, and Hao (2019) proposed a DRE-based Borda count e-

voting system called DRE-Borda. Brandl and Peters (2019) showed that the Borda mean rule is the unique social dichotomy function fulfilling neutrality, reinforcement, and the quasi-Condorcet property. Janse (2019) provided a practical explanation of the Borda count method. In addition, Kurihara (2020) used the concept of desirability of alternatives to the classic Borda scoring system.

2.3 MCDM & Voting Systems

Hwang and Lin (1987) believe that, in the process of choosing a position or a candidate, multiple criteria appear in each voter's mind. Since, some MCDM approaches have been used to voting system in the past. For instance, Stein, Mizzi, and Pfaffenberger (1994) used a ranked voting system combined with a set of point values assigned to the various ranks. So, the winner is the one with the highest total points. Fraser and Hauge (1998) applied an approval-voting concept to MCDM problems. Jimenez and Polasek (2003) proposed a multi-criteria framework for the new democratic era, e-democracy and knowledge.

Laukkanen, Palander, and Kangas (2004) used a multi-criteria decision support method based on voting theory, called multi-criteria approval (MA), to wood supply chain management in a forest area owned by the state of Finland. Liu and Hai (2005) introduced a new voting approach based on the use of Saaty's analytic hierarchy process (AHP) method that was developed to assist in multi-criteria decision-making problems. Hajimirsadeghi and Lucas (2009) extended TOPSIS for group decision making with linguistic quantifiers and concept of majority opinion. Soltanifar and Lotfi (2011) used a voting AHP method for discriminating between efficient decision making units in data envelopment analysis (DEA). Cheng and Deek (2012) suggested a mind line to studying and using voting in Group Decision Support System (GDSS). Almedia and Nurmi (2015) presented some features related to an MCDM model for aiding the choice of a voting procedure for a business organization decision problem.

Tajvidi-Asr, Hayati, Rafiee, Ataei, and Jalali (2015) selected the proper support system for Beheshtabad water transporting tunnel using SAW, TOPSIS and LA methods by considering of effective attributes. So, the optimum support system is suggested using aggregating techniques (the ranks mean, Borda and Copeland method) that is economically and safety suitable. Soltanifar (2017) presented a method for analyzing Group AHP with an unequal level of decision-makers using preferential voting system. Alguliyev, Alguliyev, and Yusifov (2019) proposed an MCDM model for the selection of candidates in e-voting environment. In addition, Azadfallah (2019) applied a new MCDM approach (particularly, AHP-based model) to solve the voting systems problem, in which

voters are classified into several groups with different importance level. So, the group with higher importance level may have a greatest effect and vice-versa.

In sum, unlike previous related works, in this paper, we proposed three new approaches for ranking candidates in voting system.

3. Research Design

According to the viewpoint proposed by Bouyssou, Marchant, and Perny (2009) the diversity of voting systems actually used in the world shows that this problem is still important. On the other side, Bouyssou, Marchant, Pirlot, Tsoukias, and Vincke (2006) believe that, in social choice theory, and more particularly in voting theory, a society needs to choose a candidate from a set of candidates. The choice of the candidate is, in most cases, based on the preferences of the voters.

This problem bears a striking similarity to the multiple criteria decision support problem in which a client needs to choose an alternative, based on preferences on different dimensions. In multiple criteria decision support, the client plays the role of society; criteria play the role of the voters, and alternatives, the role of the candidates. Therefore, voting systems is believed that to be a MCDM problem (Azadfallah, 2019).

According to Alam, Mezbahuddin, and Shoma (2015), the election result can be predicted before the actual outcome using a prediction method. Versus, a Markov chain is commonly used in stock market analysis, manpower planning, and in many other areas because of its efficiency in predicting long run behavior (Zakaria, Othman, Sokkalingam, Daud, Abdullah, & Kadir, 2019). In addition, according to Egecioglu and Giritligil (2011), the Borda rule is one of the most studied voting procedures in the social choice theory literature. Versus, one of the voting system shortcomings is that it is assumed there is no preference among voters (Azadfallah, 2019). Moreover, according to Dizaji and Khanmohammadi (2016), one of the most common ways of MCDM is TOPSIS. Versus, Egecioglu and Giritligil (2011) believe that, a voting rule solves the collective decision problem where voters must jointly choose one among a number of possible candidates (alternatives) on the basis of reported ordinal preferences. On the other side, according to Yue (2013), it is worthwhile to examine different models from different perspectives. Therefore, in this paper, we try to see all aforementioned factors together.

To conclude, in this paper, voting systems have been considered from various visions. So, a new mind line is proposed that is an enhancement over the current approach.

3.1 Proposed Method

In the following, the conventional (particularly, Markov chain, Borda's function, and TOPSIS method with interval data) and extended approach, and its characteristics are given.

- **The conventional method**

- The Markov Chain

To predict the future state, it is necessary to identify the initial state and transition probabilities fixed from the system. There are several techniques for predicting the future state, which in this paper uses matrix multiplication approach. Further, the matrix multiplication is a simple method for predicting the state of the Markov system for future periods. By having the initial state of matrix multiplication can be used for prediction system at time n (Alipoor Talemi, Jahanbani, & Heidarkhani, 2013):

A) The state of the system:

First system state at time n is showed by a one-dimensional matrix to name of vector:

$$P(n) = \{P1(n), P2(n)\}$$

That in this relation $P(n) = \text{value vector}(n).P1$

$$P1(n) = \text{the probability that system at time } n \text{ be in state} \quad (1)$$

$$P2(n) = \text{the probability that system at time } n \text{ be in state} \quad (2)$$

If we suppose that system at time n be in state 1 Then $P1(n) = [1, 0]$

If we suppose that system at time n be in state 2 Then $P2(n) = [1, 0]$

It is important to note here that, if there is a system with more than two states, not necessary that the system in the initial state is only one of the states and may be more than one state but in any state vector sum must always be equal to one. For instance, the state Vector for a system of three cases may be [0.1 and 0.7 and 0.2].

B) Matrix of transition probabilities (P):

Transition probabilities matrix is shown as following

$$P = \begin{matrix} & \begin{matrix} 1 & 2 \end{matrix} \\ \begin{matrix} 1 \\ 2 \end{matrix} & \begin{matrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{matrix} \end{matrix}$$

In this matrix:

P = Transition probabilities matrix
 P_{ij} = Transition probabilities of system from state i to state j .

C) Prediction of the future state:

To calculate the probability that system at time $(n+1)$ be state j , we use as follows relationship:

$$P(n+1) = P(n).P$$

That this relationship: $P(n+1)$ = State probability vector at time $n+1$ (one time period later)

$P(n)$ = State probability vector at time (n) (time period)
 P = Transition probabilities matrix

According to above relationship can be calculated transition probabilities in several periods later in form of simple.

$$P(n+2) = P(n+1).P$$

$$P(n+3) = P(n+2).P$$

A general relationship obtained from these relationships is as follows:

$$P(n) = P(0).P^n \quad (3)$$

That in this equation:

$P(n)$ = State probability vector in time period n^{th} ,
 $P(0)$ = probability vector in time period zero,
 P^n = power of n^{th} in transition probabilities matrix,
 n = Number of time periods for which it is predicted.

Moreover, in the Markov process often by more n (in long term) value vector tends to fixing state (stable state). As to achieve its period multiplying the state vector in transition probabilities matrix is equal to transition probabilities matrix in periods later that this state is called stable state.

If stable value show by π symbol, instable state, the state vector will be in terms of decimal values as follow:

$$\pi = [\pi_1, \pi_2]$$

In this relationship:

Π = State probability vector (as relative amounts):

$$\pi_1 = \text{amount of state 1} \quad (4)$$

$$\pi_2 = \text{amount of state 2} \quad (5)$$

Since in stable state conditions isn't important time period and values are independent of time, multiplication of state vector in transition matrix a vector in stable state will be same as state vector. Therefore, stable state values can be determined based on the following algebraic method:

$$\pi = \pi.P \rightarrow \pi = [\pi_1, \pi_2]^* \rightarrow \begin{cases} \pi_1 = \pi_1.P_{11} + \pi_2.P_{21} \\ \pi_2 = \pi_1.P_{12} + \pi_2.P_{22} \end{cases}$$

On the other hand, sum of probability states must be equal to one: $\pi_1 + \pi_2 = 1$.

- o Borda count – social choice method

According to Srdjevic, Srdjevic, and Medeiros (2017), Preferential voting methods from the SC (social choice) theory exclusively use ordinal preference information contained in the preference table (table 2), created by collecting ballots (in real elections). A created preference table usually has the following properties. The size of the table is $M.N$, where M is the number of individuals and N is the number of possible alternatives (choices). Each row represents the ranking of alternatives performed by one individual. If j is the best alternative for individual i , then the rank number is $r_{ij} = 1$; if j is the second-best alternative, then $r_{ij} = 2$, and so on; if alternative j is the worst one, then $r_{ij} = N$.

Table 1. Preference table

	Alt. 1	Alt. 2	...	Alt. J	...	Alt. N
Indiv. 1	r_{11}	r_{12}	...	r_{1j}	...	r_{1N}
Indiv. 2	r_{21}	r_{22}	...	r_{2j}	...	r_{2N}
...
Indiv. i	r_{i1}	r_{i2}	...	r_{ij}	...	r_{iN}
...
Indiv. M	r_{M1}	r_{M2}	...	r_{Mj}	...	r_{MN}

In Borda count, each alternative gets 1 point for each last place vote. Similarly, 2 points for each next-to-last point vote and so on up to N points. The alternative with the leading total point wins the election and is declared to be the social choice.

For each r_{ij} in the preference schedule, a number

$$q_{ij} = N - r_{ij} + 1 \quad (6)$$

is allocated by the above instructions, and the total score for alternative j is given as

$$Q_j = \sum_{i=1}^M q_{ij} = \sum_{i=1}^M (N - r_{ij} + 1) = M(N+1) - \sum_{i=1}^M r_{ij} \quad (7)$$

The alternative j^* with the highest Q can be selected as the winner, i.e. Social choice:

$$Q_{j^*} = \max_{1 \leq j \leq N} Q_j. \quad (8)$$

- o TOPSIS with interval data

In Jahanshahloo, Lotfi, and Izadikhah (2006), an interval extension of original TOPSIS method was

proposed. This approach may be illustrate as follow.

1. Calculate the normalized decision matrix. The normalized value \bar{n}_{ij} is calculated as:

$$\bar{n}_{ij}^l = x_{ij}^l / \sqrt{\sum_{j=1}^m (x_{ij}^l)^2 + (x_{ij}^u)^2}, \quad j=1, \dots, m, \quad i=1, \dots, n, \quad (9)$$

$$\bar{n}_{ij}^u = x_{ij}^u / \sqrt{\sum_{j=1}^m (x_{ij}^l)^2 + (x_{ij}^u)^2}, \quad j=1, \dots, m, \quad i=1, \dots, n \quad (10)$$

2. Calculate the weighted normalized interval decision matrix. The weighted normalized value \bar{v}_{ij} is calculated as:

$$\bar{v}_{ij}^l = w_i \bar{n}_{ij}^l, \quad j=1, \dots, m, \quad i=1, \dots, n, \quad (11)$$

$$\bar{v}_{ij}^u = w_i \bar{n}_{ij}^u, \quad j=1, \dots, m, \quad i=1, \dots, n, \quad (12)$$

Where w_i is the weight of the i_{th} attribute or criterion, and $\sum_{i=1}^n = 1$.

3. Determine the positive ideal and negative ideal solution.

$$\bar{A}^+ = \{\bar{v}_1^+, \dots, \bar{v}_n^+\} = \{(max_j \bar{v}_{ij}^u / i \in I), (min_j \bar{v}_{ij}^l / i \in J)\}, \quad (13)$$

$$\bar{A}^- = \{\bar{v}_1^-, \dots, \bar{v}_n^-\} = \{(min_j \bar{v}_{ij}^l / i \in I), (max_j \bar{v}_{ij}^u / i \in J)\}, \quad (14)$$

Where I is associated with benefit-type criteria and J is associated with cost-type criteria.

4. Calculate the separation measures, using the n-dimensional Euclidean distance. The separation of each alternative from the solution is given as:

$$\bar{d}_j^+ = \{\sum_{i \in I} (\bar{v}_{ij}^l - \bar{v}_i^+)^2 + \sum_{i \in J} (\bar{v}_{ij}^u - \bar{v}_i^+)^2\}^{1/2}, \quad j = 1, \dots, m. \quad (15)$$

Similarly, the separation from the negative ideal solution can be calculated as:

$$\bar{d}_j^- = \{\sum_{i \in I} (\bar{v}_{ij}^u - \bar{v}_i^-)^2 + \sum_{i \in J} (\bar{v}_{ij}^l - \bar{v}_i^-)^2\}^{1/2}, \quad j = 1, \dots, m. \quad (16)$$

5. Calculate the relative closeness to the ideal solution. The relative closeness of the alternative A_j with respect to \bar{A}^+ is defined as:

$$\bar{R}_j = \bar{d}_j^- / (\bar{d}_j^+ + \bar{d}_j^-), \quad j = 1, \dots, m. \quad (17)$$

Obviously, an alternative A_j is closer to the \bar{A}^+ and farther from \bar{A}^- as \bar{R}_j approaches to 1.

(6) Rank the preference order. According to the closeness coefficient, we can identify the ranking of all alternatives and select the best one from possible alternatives.

• **The extended approach (Proposed model)**

- The Markov chain-based approach

In many researches, Markov chain is often used for predicting long run attributes but not for discover the efficient electoral strategy for each candidate. The present

paper points to this problem. Steps of proposed approach (for election result prediction by pre-election preference information to identify the efficient electoral strategy for each candidate) explained as follows.

Step1. Gaining votes received by each of candidates for first rank (or place) by same voters from diverse political wings in election (i.e., republicans, democrats, etc.), before and after the present of program by candidates (for instance, a few weeks/ months before the election).

Step2. Provide the transition matrix for people voting for candidates to identify voting changes from candidate to candidate after normalizing (column) such that; for $\{a_1, a_2, a_3\}$, $P_{a1} = a_1 / (a_1 + a_2 + a_3)$, $P_{a2} = a_2 / (a_1 + a_2 + a_3)$, and $P_{a3} = a_3 / (a_1 + a_2 + a_3)$.

Step3. Determine the initial-state probabilities into an initial-state vector.

Step4. Use the transition matrix and initial-state vector (multiplication of state vector in transition matrix) to reach the stable state condition (in other words, the process has convergence in iterative transformation and the calculation have been stabled).

Step5. Use the obtain results (competitor analysis) to discover recommendations and strategies to provide the efficient electoral strategy (recommended strategies, include, 1. Continue the current situation (status-quo) strategy, 2. Political advertising or electoral campaign strategy, and 3. Exit strategy) for each candidate [based on the set threshold, as discussed latter in this paper].

- The Borda's function-based approach

As noted earlier, according to Egecioglu and Giritligil (2011), the Borda rule is one of the most studied voting procedures in the social choice theory literature. However the efficiency of Borda's function is undeniable, there is a significant limit for it. This method is not capable of taking the different importance weights to voters. In this paper, to remove this constraint, a new Borda's function-based approach is highlighted. Whilst, each voter who has more knowledge, expertise and experience on a special field (political, economic, etc.) will have the highest score for it, and vice-versa. In this paper, we use Borda's function to compute the candidate scores in the election, but we alter the procedure of calculating the traditional Borda's function to the extended model. So, in the proposed method, first we assume that voters are categorized into various levels (in which the vote of voters in a higher level is more important than the ones in a lower level (table 2).

Table 2. Classifying voters into several levels

Voter	Level	Voter importance level
1	1	$1 \geq 2 \geq \dots \geq m$
2	2	...
⋮	⋮	⋮
⋮	⋮	⋮
⋮	⋮	⋮
K	r_{MI}	⋮

Here, we assume that the voter importance levels follow a linear function (y):
 $y = \beta(t_i)$ (18)

Where β and t_i ($i=1, 2, \dots, m$), respectively, are the voter importance level (determined by super decision maker [supra DM], as discussed latter in this paper]) and level number. $B \geq 1$ and integer as well.

It is also worth noting here that, according to Hwang and Lin (1987), Borda's function is homogeneous, monotonic, Pareto optimal, anonymous, and neutral. Before continuing, it is necessary to define these characteristics.

• **Neutrality**

This property says that the social choice will be reversed if every voter reverses his/her vote. In other words, the system should treat all candidates equally.

• **Anonymity**

This property is in accordance with the principle of one person-one vote. Further, the system gives equal weight to each voter.

• **Monotonicity**

This property means that, if a voter moves x upward in his ranking and leaves the relative standing of the others unchanged, then candidate x will stand at least as well relative to each other candidate as before. It also holds if several voters make changes in x 's favor.

• **Homogeneity**

This property explains that a voter indifferent among several candidates can be replaced by several fractional voters holding symmetric views on them, for example, if a voter is indifferent between x and y , he/she is replaced by two voters, each with same preferences as the original except that one prefers x to y and the other y to x .

Finally, the meaning of *Pareto Optimality* is that if every voter thinks x is better than y (or at least as good as),

then so does society. The Pareto optimality is also referred to as unanimity (Hwang & Lin, 1987).

Notice:

- A. Regarding the power relation system amongst the DMs (in other words, voters importance level), one of them may be a supra-DM, who usually has a hierarchical position in the organization's structure that is higher than that of the other DMs (Almedia, Morais, and Nurmi, 2019).
- B. To state the obvious, the proposed approaches violate the neutrality and anonymity characteristics.

TOPSIS with interval data-based approach

TOPSIS is a practical and useful technique for ranking and selection of a number of externally determined alternatives through distance measures (Shih, Shyur, and Lee, 2007). This method was established by Hwang and Yoon (1981). While, Jahanshahloo, Lotfi, and Izadikhah (2006) extended TOPSIS for decision making problems with interval data.

On the other side, a voting rule solves the collective decision problem where voters must jointly choose one among a number of possible candidates (alternatives) on the basis of reported ordinal preferences (Egecioglu and Giritligil, 2011). Meanwhile, the TOPSIS with interval data method is unable to manage appropriately ordinal information. Therefore, first we used the transformation of ordinal preference information to interval data developed by Wang, Greatbanks, and Yang (2005), and then solve the non-ordinal MCDM problem using the TOPSIS with interval data method. In the following, we briefly set known the above methods as follows.

• **Converting Ordinal Data to Interval Number model**

According to Wang, Greatbanks, and Yang (2005), method for strong ordinal preference information $y_{r1} > y_{r2} > \dots > y_{rn}$, we have the following ordinal relationships after scale transformation:

$$1 \geq \hat{y}_{r1}, \hat{y}_{rj} \geq X_r \hat{y}_{r, j+1} \ (j=1, \dots, n-1) \ \text{and} \ \hat{y}_{rn} \geq \sigma_r \tag{19}$$

Where X_r is a preference intensity parameter satisfying $X_{r>1}$ provided by the DM and σ_r is the ratio parameter also provided by the DM. the resultant permissible interval for each \hat{y}_{rj} can be derived as follows:

$$\hat{y}_{rj} \in [\sigma_r X_r^{n-j}, X_r^{1-j}], \ j=1, 2, \dots, n \ \text{with} \ \sigma_r \leq X_r^{1-n} \tag{20}$$

• **Numerical example**

In order to illustrate the application of the proposed methods in this paper, three examples are given as follows.

Example 1 (For Markov-based approach)

Assume that $\{a, b, c, d\}$ be the set of candidates for a 100 voter election problem. A few weeks / months before

the election, the results of two survey (with the same voters, from different political parties, before and after the present of programs by four candidates (in step 1] are as follows (table 3).

Table 3. The voter preferences

Candidate	Votes received by four candidates for first place* **	Add				Lose				Change measure				Votes received by four candidates for first place***
		a	b	c	d	a	b	C	d	a	b	c	d	
a	29	0	3	2	5	0	7	3	1	18	3	2	5	28
b	15	7	0	0	6	3	0	4	0	7	8	0	6	21
c	23	3	4	0	3	2	0	0	3	3	4	18	3	28
d	33	1	0	3	0	5	6	3	0	1	0	3	19	23
Σ	100	-				-				-				100

Note: *.A voter chooses only his/ her favorite candidate instead of ranking them all,
 **. Before the present of program by candidate,
 ***. After the present of program by candidate.

Now consider how to obtain the elements of the transition matrix (table 4) [in step 2]. For instance, for $P_{a,a}$: $P_{a,a}=18/(18+7+3+1)=0.621$

Table 4. The transition matrix [P(0)]

-	A	b	c	d
a	0.621	0.200	0.087	0.152
b	0.241	0.533	0.000	0.182
c	0.103	0.267	0.783	0.091
d	0.034	0.000	0.130	0.576
Σ	1	1	1	1

The entry in table 4, presents the probability of transition from the state corresponding i to the state corresponding to j . according to the above table (i.e., for first candidate), the vote received that began in a , 62% will again be in first place, 24% will be in b , 10% will be in c , and 3% will be in d , respectively.

In step 3, we determine the initial-state probabilities vector. Assume that, the initial distribution indicates the actual voter (the first column in table 3) in the system, thus, the vector is as follows. For instance, for $P_a^{(0)}$: $P_a^{(0)}=29/(29+15+23+33)=0.290$

$$P^{(0)} = \begin{pmatrix} 0.290 \\ 0.150 \\ 0.230 \\ 0.330 \end{pmatrix}$$

In step 4, we use the transition matrix and initial-state vector to reach the stable state condition. Thus:

$$P(1) = \begin{pmatrix} 0.621 & 0.200 & 0.087 & 0.152 \\ 0.241 & 0.533 & 0.000 & 0.182 \\ 0.103 & 0.267 & 0.783 & 0.091 \\ 0.034 & 0.000 & 0.130 & 0.576 \end{pmatrix} \cdot \begin{pmatrix} 0.290 \\ 0.150 \\ 0.230 \\ 0.330 \end{pmatrix} = \begin{pmatrix} 0.280 \\ 0.210 \\ 0.280 \\ 0.230 \end{pmatrix}$$

As can be seen from above, after one transition, the distribution will be 28% of votes for a , 21% for b , 28% for c , and 23% for d , respectively.

Then in, transition:

- P (1) = (0.280, 0.210, 0.280, 0.230),
- P (2) = (0.275, 0.221, 0.325, 0.179),
- P (3) = (0.270, 0.217, 0.358, 0.155),
- P (4) = (0.266, 0.209, 0.380, 0.145),
- P (5) = (0.262, 0.202, 0.394, 0.142),
- P (6) = (0.259, 0.197, 0.402, 0.142),
- P (7) = (0.256, 0.193, 0.407, 0.143),
- P (8) = (0.255, 0.191, 0.410, 0.144),
- P (9) = (0.254, 0.190, 0.411, 0.145),
- P (10) = (0.253, 0.189, 0.412, 0.146),
- P (11) = (0.253, 0.188, 0.412, 0.147),
- P (12) = (0.253, 0.188, 0.412, 0.147).

Results that obtained in the eleventh iteration, identically repeated in twelfth iteration. In other words, no change, so we stop here. Further, we can see that the third candidate (c) is ranked first, and the fourth candidate (d) is ranked last (Fig. 1).

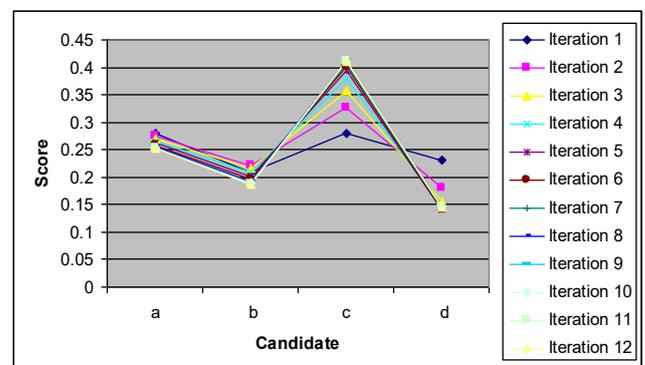


Fig. 1: The comparative results

In step 5, we use the obtain result (in previous step) to discover recommendation and strategies to provide the efficient electoral strategy for each candidate, as follows.

Here, assume that, strategies for candidate include the following:

For best-ranked candidate, continue the current situation (status-quo) strategy is suggested - For top-ranked candidate, resume the current situation strategy is suggested, If the distance between the two near candidates (best-rank candidate with other candidate) was compared with the set threshold (notice; the threshold value should be determined by expert or supra DM), while, the difference was more than the threshold.

For candidate between two best and worst rank, the political advertising (or electoral campaign) strategy is recommended - For candidate between two best and worst rank, the political promotion (or electoral campaign) strategy is suggested, if the distance between the two candidates is shorter than the set threshold, this strategy is emphasized. Nevertheless, most candidates between the best rank and worst rank candidates are expected to be here.

For worst ranked candidate, the exit strategy is suggested - For worst ranked candidate, the exit strategy is suggested, if the distance between the two near candidate (worst rank candidate with other candidate) is greater than the set threshold. If not, it is subject to strategy number 2 (the political advertising/ or electoral campaign strategy).

In this section, this resultant value [$P(12) = (a=0.253, b=0.188, c=0.412, d=0.147)$] will be compared with 0.060, i.e., threshold value. thus,

For best-ranked candidate:

Distance (Best-ranked candidate score - other candidate score) = $(c-a) = |0.412-0.253| = 0.159 > 0.060$.

Because the difference is greater than the threshold value, the first strategy is recommended for candidate with the best rank (in other words, candidate of c). *Notice;* because the difference between the two nearest candidates (best rank candidate with other candidate) is greater than the threshold, calculation is not performed for the rest candidates.

For worst-ranked candidate:

Distance (worst ranked candidate score - other candidate score) = $(d-b) = |0.147-0.188| = 0.041 > 0.060$.

Because the difference is not greater that the threshold value, the last strategy (in other words, exit strategy) is suggested for two candidates (candidates of d and b).

For rest of candidate (here, candidate of a):

Distance (rest of candidate score - worst ranked candidate score) = $(a-c) = |0.253-0.412| = 0.159 > 0.060$.

Distance (rest of candidate score - best ranked candidate score) = $(a-b) = |0.253-0.188| = 0.065 > 0.060$.

Because the difference between a, b and a, c is greater than the threshold value respectively, then this candidate (a) does not belong to the first and third strategies and should think about the second strategy (the political advertising/ or electoral campaign strategy). As can be concluded from the above-mentioned consequences, the proposed approach applies the information of shifts in ideas after the presentation of programs by the candidates, to achieve the efficient electoral strategy for each candidate. Therefore, the first, second, and third strategy (continue the current situation, the political advertising or electoral campaign, and exit strategy) is recommended for candidate of c, a , and $b \& d$, respectively.

Example 2 (For Borda's function-based approach)

Assume that $\{a, b, c, d\}$ be the set of candidates for a 30 voter election problem. For the first step, the list of the voter's preferences, which is called a profile, classifying voters, the modified voter preferences, and summary of voter's preferences, is (table 5-8):

Table 5. The voter preferences

Voter number	Number of voter	Preference
1, 2, 3, 6, 12, 15, 18, 21, 26, 29	10	$a > b > c > d$
4, 5, 8, 9, 10, 13, 23, 30	8	$b > a > d > c$
7, 14, 19, 22, 27	5	$c > a > d > b$
11, 16, 17, 20, 24, 25, 28	7	$d > b > c > a$
Σ	30	-

Table 6. Classifying voters into several categories

Voter number	Number of voter	Level (category)	Voter importance level*
3, 7, 22	3	1	$\beta=3$, then $y=3t_i$
1,5,18, 21, 30	5	2	$\beta=2$, then $y=2t_i$
2, 4, 6, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 19, 20, 23, 24, 25, 26, 27, 28, 29	22	3	$\beta=1$, then $y=1t_i$
Σ	30	-	-

Note:*.notice here (i.e., for voter in category 1; voter 3), preference from $a > b > c > d$, convert to $a > b > c > d + a > b > c > d + a > b > c > d$.

Table 7. The revised voter preferences (based on Equation [18] and table 5-6)*

Voter number	Number of voter	Preference
1, 2, 3, 6, 12, 15, 18, 21, 26, 29	Voter no. 3 = 3t _i Voter no. 1, 18, 21 = 2t _i Voter no. 2, 6, 12, 15, 26, 29 = 1t _i	a>b>c>d
4, 5, 8, 9, 10, 13, 23, 30	Voter no. - = 3t _i Voter no. 5, 30 = 2t _i Voter no. 4, 8, 9, 10, 13, 23 = 1t _i	b>a>d>c
7, 14, 19, 22, 27	Voter no. 7, 22 = 3t _i Voter no. - = 2t _i Voter no. 14, 19, 27 = 1t _i	c>a>d>b
11, 16, 17, 20, 24, 25, 28	Voter no. - = 3t _i Voter no. - = 2t _i Voter no. 11, 16, 17, 20, 24, 25, 28 = 1t _i	d>b>c>a
Σ	41	-

Note:*. Notice that the number of voter preferences increased from 30 to 41, by the proposed method.

Table 8. The summary of voter's preferences

Voter number	Number of voter	Preference
1, 2, 3, 6, 12, 15, 18, 21, 26, 29	15	a>b>c>d
4, 5, 8, 9, 10, 13, 23, 30	10	b>a>d>c
7, 14, 19, 22, 27	9	c>a>d>b
11, 16, 17, 20, 24, 25, 28	7	d>b>c>a
Σ	41	-

Then, solved current problem (table 8) using a conventional Borda's function algorithm. In continuation, the Borda count for candidate of *a* is given by:

$$(a) = (\text{No. 1}^{\text{st}} \text{ place votes}) * 4 + (\text{No. 2}^{\text{nd}} \text{ place votes}) * 3 + (\text{No. 3}^{\text{rd}} \text{ place votes}) * 2 + (\text{No. 4}^{\text{th}} \text{ place votes}) * 1 = (4 * 15) + (3 * 10) + (3 * 9) + (1 * 7) = 124,$$

Similarly,
 (b)= 115,
 (c)=90,
 (d)= 81.

From the above results, it can be easily derived that, the implied ranking candidate is as follows.
 $a > b > c > d$

As can be seen, the first candidate (*a*) has the best performance, so the winner with this method is *a*. The question typically asked is which candidate will win if the conventional Borda method is used?

With no intension to describe the whole procedure, we shall only point to the final results (based on table 5), thus:

$$(a) = 86,
 (b) = 88,
 (c) = 62,
 (d) = 64.$$

From the above results, it can be concluded that, the ranking is as follows.

$$b > a > d > c$$

Therefore, the second candidate (*b*) has the best performance, so the winner with this method is *b*.

A comparison of test results is given in table 9

Table 9. The comparative results

Method	Preference
Conventional method*	$b > a > d > c$
Proposed method**	$a > b > c > d$

Note:*. Based on voter preference matrix,
 **. Based on revised voter preference matrix.

As can be seen in table 9, the different between two models are crystal clear. This difference is due to the voter's weights considered. To say it better, voter's weights impact could greatly enhance the decision making process. So, *a* become the proper candidate instead of *b*.

Example 3 (For interval TOPSIS -based approach)

Assume that a total of six candidates are presented based on four criteria (table 9), as follows.

Table 10. The collective decision matrix (ranked candidates based on voter's consensus)*

Criteria \ Candidate	C ₁ **	C ₂	C ₃	C ₄
A	1	4	3	4
B	3	2	2	7
C	5	3	7	6
D	2	7	6	1
E	6	5	1	5
F	4	6	4	2
G	7	1	5	3

Note:*. Assume that the weight of criteria by supra DM is set as follows, C₂>C₄>C₁>C₃,

** Cost-type criteria.

In this step, we transform the matrix rating from ordinal format to the form of interval number (table 10-11), by Eq. (20), as follows.

$\hat{y}_{ij} \in [\sigma_r X_r^{n-j}, X_r^{1-j}]$, $j=1, 2, \dots, n$ with $\sigma_r \leq X_r^{1-n}$, further, according to the view point proposed by Wang, Greatbanks, and Yang (2005), $X=1.12$ and $\sigma=0.1$, respectively.

Table 11. Transform matrix

-	Rank	Lower bound	Upper bound	Interval value
For preferences	1	$=0.1*(1.12)^{7-1}=0.197$	$=(1.12)^{1-1}=1$	[0.197, 1]
	2	$=0.1*(1.12)^{7-2}=0.176$	$=(1.12)^{1-2}=0.893$	[0.176, 0.893]
	3	$=0.1*(1.12)^{7-3}=0.157$	$=(1.12)^{1-3}=0.797$	[0.157, 0.797]
	4	$=0.1*(1.12)^{7-4}=0.140$	$=(1.12)^{1-4}=0.712$	[0.140, 0.712]
	5	$=0.1*(1.12)^{7-5}=0.125$	$=(1.12)^{1-5}=0.636$	[0.125, 0.636]
	6	$=0.1*(1.12)^{7-6}=0.112$	$=(1.12)^{1-6}=0.567$	[0.112, 0.567]
	7	$=0.1*(1.12)^{7-7}=0.100$	$=(1.12)^{1-7}=0.507$	[0.100, 0.507]
For criteria weights	1	$=0.1*(1.12)^{4-1}=0.140$	$=(1.12)^{1-1}=1$	[0.140, 1]
	2	$=0.1*(1.12)^{4-2}=0.125$	$=(1.12)^{1-2}=0.893$	[0.125, 0.893]
	3	$=0.1*(1.12)^{4-3}=0.112$	$=(1.12)^{1-3}=0.797$	[0.112, 0.797]
	4	$=0.1*(1.12)^{4-4}=0.100$	$=(1.12)^{1-4}=0.712$	[0.100, 0.712]

Table 12. The interval decision matrix *

Criteria Alternative	C ₁ **		C ₂		C ₃		C ₄	
	x _{1j} ^l	x _{1j} ^u	x _{2j} ^l	x _{2j} ^u	x _{3j} ^l	x _{3j} ^u	x _{4j} ^l	x _{4j} ^u
a	0.197	1	0.140	0.712	0.157	0.797	0.140	0.712
b	0.157	0.797	0.176	0.893	0.176	0.893	0.100	0.507
c	0.125	0.636	0.157	0.797	0.100	0.507	0.112	0.567
d	0.176	0.893	0.100	0.507	0.112	0.567	0.197	1
e	0.112	0.567	0.125	0.636	0.197	1	0.125	0.636
f	0.140	0.712	0.112	0.567	0.140	0.712	0.176	0.893
g	0.100	0.507	0.197	1	0.125	0.636	0.157	0.797

Note:*. The criteria weight based on table 10 is set as follows, C₁ = [0.112, 0.797], C₂ = [0.140, 1], C₃ = [0.100, 0.712], C₄ = [0.125, 0.893], **. Cost-type criteria.

Next, when the TOPSIS method with interval number (Eq. 9-17) is applied, the following values are derived (table 12-17 and figure2).

Table 13. The interval normalized decision matrix (\bar{n}_{ij})

Criteria Alternative	C ₁		C ₂		C ₃		C ₄	
	x _{1j} ^l	x _{1j} ^u	x _{2j} ^l	x _{2j} ^u	x _{3j} ^l	x _{3j} ^u	x _{4j} ^l	x _{4j} ^u
a	0.098	0.495	0.069	0.353	0.084	0.106	0.069	0.353
b	0.078	0.395	0.087	0.442	0.94	0.479	0.050	0.251
c	0.062	0.315	0.078	0.395	0.54	0.272	0.055	0.281
d	0.087	0.442	0.050	0.251	0.060	0.304	0.098	0.495
e	0.055	0.281	0.062	0.315	0.106	0.536	0.062	0.315
f	0.069	0.353	0.055	0.281	0.075	0.382	0.087	0.442
g	0.050	0.251	0.098	0.495	0.067	0.341	0.078	0.395

Table 14. The interval weighted normalized decision matrix (\bar{v}_{ij})*

Criteria Alternative	C ₁		C ₂		C ₃		C ₄	
	x _{1j} ^l	x _{1j} ^u	x _{2j} ^l	x _{2j} ^u	x _{3j} ^l	x _{3j} ^u	x _{4j} ^l	x _{4j} ^u
a	0.011	0.395	0.010	0.353	0.008	0.075	0.009	0.315
b	0.009	0.315	0.012	0.442	0.009	0.341	0.006	0.224
c	0.007	0.251	0.011	0.395	0.005	0.194	0.007	0.251
d	0.010	0.353	0.007	0.251	0.006	0.216	0.012	0.442
e	0.006	0.224	0.009	0.315	0.011	0.382	0.008	0.281
f	0.008	0.281	0.008	0.281	0.008	0.272	0.011	0.395
g	0.006	0.200	0.014	0.485	0.007	0.243	0.010	0.353

Note:*. Based on table10; W_j = (C₁ = [0.112, 0.797], C₂ = [0.140, 1], C₃ = [0.100, 0.712], C₄ = [0.125, 0.893]).

Table 15. The positive and negative ideal solution (\bar{A})

-	C ₁	C ₂	C ₃	C ₄
\bar{A}^+	0.006	0.495	0.382	0.442
\bar{A}^-	0.395	0.007	0.005	0.006

Table 16. Distance of each alternative from the positive ideal solution (\bar{d}_j^+)

\bar{d}_1^+	\bar{d}_2^+	\bar{d}_3^+	\bar{d}_4^+	\bar{d}_5^+	\bar{d}_6^+	\bar{d}_7^+
0.846	0.810	0.791	0.828	0.782	0.800	0.773

Table 17. Distance of each alternative from negative ideal solution (\bar{d}_j^-)

\bar{d}_1^-	\bar{d}_2^-	\bar{d}_3^-	\bar{d}_4^-	\bar{d}_5^-	\bar{d}_6^-	\bar{d}_7^-
0.606	0.706	0.629	0.665	0.681	0.669	0.753

Table 18. Closeness coefficient (\bar{R}_j)

\bar{R}_1	\bar{R}_2	\bar{R}_3	\bar{R}_4	\bar{R}_5	\bar{R}_6	\bar{R}_7
0.417	0.465	0.443	0.446	0.465	0.455	0.493

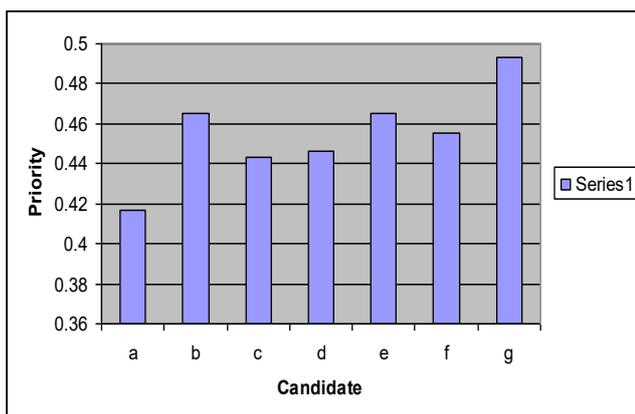


Fig. 2: The comparative results

From the above results, it can be concluded that, the priorities is as follow:

$$g [0.493] > e [0.465] \approx b [0.465] > f [0.455] > d [0.446] > c [0.443] > a [0.417]$$

Therefore, the winner with this method is seventh candidate (g). While, 1st candidate (a) have very bad performance.

4. Discussion

In example 1; we can find the $P(I) = (a=0.280, b=0.210, c=0.280, d=0.230)$. This priority is different from

that of the eleventh iteration ($P(11) = (a=0.253, b=0.188, c=0.412, d=0.147)$). this difference is due to the present of program by candidate considered. So, c (0.412) becomes the suitable candidate instead of a and c (0.280, 0.280, respectively). In continuation, we use the information of shifts in opinions after the present of programs by candidates, to achieve the efficient electoral strategy (by a set of threshold) for each candidate (table 18).

Table 19. The summary of results

Candidate	Recommended strategy	Description (action plan)
a	Strategy 2 (the political advertising/ electoral campaign)	<ul style="list-style-type: none"> Promotion on social media Promotion by radio, TV, newspaper, E-mail lists, sent text message, etc. Combating fake news Wrote about your experience and skills on a personal website or blog Create an opportunity for on-line discussion Increasing their advertising budget.
b	Strategy 3 (exit strategy)	<ul style="list-style-type: none"> Full exit from election and planning for the next elections Coalition and support a candidates who has closer goals and aspiration with you.
c	Strategy 1 (continue the current situation)	<ul style="list-style-type: none"> Continuation of previous programs (or strategies) because of their effectiveness.
d	Strategy 3 (exit strategy)	<ul style="list-style-type: none"> Full exit from election and planning for the next elections Coalition and support a candidates who has closer goals and aspiration with you.

In example 2; as can be seen in table 8, the differences between two models are clear. The candidate ranking is $a > b > c > d$. this differs from that of the conventional Borda's function model ($b > a > d > c$). Because the DMs weights (or voters weights) are considered into the proposed method. In this situation, a will be the suitable candidate instead of b .

In example 3; according to the results of table 17 (notice, first the ordinal data were transformed into interval number and then input them into the model), we can find

the priority is $7 [0.493] > 5 [0.465] \approx 2 [0.465] > 6 [0.455] > 4 [0.446] > 3 [0.443] > 1 [0.417]$. In compare to conventional ordinal method (i.e., Borda, Mean rank, Copeland, etc.), the proposed method is this paper has attractive advantage. So, the results are presented in both ordinal and cardinal form.

On the other side, according to Mohaghar, Kashef, and Khanmohammadi (2014), in order to find a solution to the problem, a variety of authors have tried to combine two or more techniques through shifting the solution in a specific stage to another technique or using results of one as input of another based on a logical idea. These innovative approaches can both cover the weaknesses of different techniques and pave the way to benefit from the advantages of all involved techniques simultaneously. The proposed model addresses this problem. hence, first the ordinal data were transformed into interval number (by Wang et al. [2005] transformation formula) and then input them into the MCDM model (particularly, TOPSIS with interval data, by Jahanshahloo et al. [2006]). Therefore, voters and candidates can more assurance o the results by using a systematic model. Finally, findings in this paper confirm the effectiveness of proposed methods.

5. Conclusion

According to Aghayi and Tavana (2019), obtaining a group ranking or a winning candidate from individual's preferences on a set of alternatives is an important group decision problem with social choice and voting system implications. Nevertheless, several solutions have been proposed for solving this problem too. In this paper voting system and some famous models have been studied from different perspectives. So, we propose three new approaches (1. election result prediction by pre-election preference information using Markov chain model [to identify the efficient electoral strategy for each candidate]. 2. Improved Borda's function method using the weights of decision makers [or voters]. And 3.a new interval TOPSIS-based approach using ordinal set of preferences [so, data is ordinal form that first convert to interval value and then input them into the conventional interval TOPSIS model]) for ranking candidates in voting systems. In continuation, in order to illustrate the application of the proposed methods in this paper, three examples are given. So, in example 1, we use the information of shifts in opinions after the presentation of programs by the candidates (from $P(I) = (a=0.280, b=0.210, c=0.280, d=0.230)$ to $P(II) = (a=0.253, b=0.188, c=0.412, d=0.147)$), to achieve the efficient electoral strategy (by a set threshold) using Markov chain model for each candidate. In brief, the first, second, and third strategy (continue the current situation, the political advertising or electoral campaign, and exit strategy) is recommended for candidate of c , a , and b & d ,

respectively. In example 2, as can be seen in table 8, we can find the ranking $a > b > c > d$. this ranking is different from that the conventional approach (Borda's function method) $b > a > d > c$ respectively. This difference is due to the DMs weights (or voter's weights) considered. In addition, in example 3, we first transform ordinal preferences to interval number and then input them into the conventional interval TOPSIS model. Nevertheless, as can be seen in table 17, we can find the priority is $g [0.493] > e [0.465] \approx b [0.465] > f [0.455] > d [0.446] > c [0.443] > a [0.417]$.in compare to conventional ordinal method (i.e., Borda, Rank mean, Copland, etc.), the proposed method in this paper has attractive advantage. So, the results are presented in both ordinal and cardinal form.

We think that, the attractiveness of the proposed models is that they are direct and ensures transparency in the decision process (because of the proposed methods is a novel approach sourcing from Markov chain model, Borda's function and interval TOPSIS method), it sustains group/ collective decision making problems, and do not require the modify the conventional methods. On the other hand, it does not need the extra data from DM or voters, and the results can give more assurance by applying systematic model. As a general fact, this paper offers a framework for reducing the wrong option winner risks associated with voting systems.

In sum, the finding in this paper confirms the effectiveness of propose methods. However, the proposed approaches have some notable limitations; for example, it requires a rather long calculation (in the data pre-processing step). In addition, proposed methods (particularly, transformation formula from ordinal form to interval form), has been developed for the Data Envelopment Analysis (DEA) environment. Since, this could cause some bias in the final results. So, more studied are needed. Furthermore, according to Bouyssou, Marchant, and Perny (2009), the many results obtained in social choice theory are valuable for multi-criteria decision aiding. Hence, these results can be used in both sections. Finally, it is expected that the new approach proposed in this paper can play an important role in the studies and applications of the Multi Criteria Group Decision Making (MCGDM) and voting systems. So, the MCGDM and voting systems problem can be solved effectively and efficiently.

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