

Gupta Transform Approach to the Series RL and RC Networks with Steady Excitation Sources

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Abstract— The analysis of electric networks circuits is an essential course in engineering. The response of such networks is usually obtained by mathematical approaches such as Laplace Transform, Calculus Approach, Convolution Theorem Approach, Residue Theorem Approach. This paper presents a new integral transform called Gupta Transform for obtaining the complete response of the series RL and RC networks circuits with a steady voltage source. The response obtained will provide electric current or charge flowing through series RL and RC networks circuits with a steady voltage source. In this paper, the response of the series RL and RC networks circuits with steady excitation source is provided as a demonstration of the application of the new integral transform called Gupta Transform.

Keywords — Gupta Transform; Series RL and RC Networks circuits; Response.

1. Introduction

The electric circuit of the series RL network consists of two passive electric elements: an inductor L and a resistor R , connected in series with a steady voltage source and the electric circuit of the series RC network consists of two passive electric elements: a capacitor C and a resistor R , connected in series with a steady current source. Such networks are used as a tuning or resonant circuit in the radio and television set to a particular frequency band from the wide range of frequency components, or in the chokes of luminescent tubes [1-4]. The Gupta Transform has been proposed recently by the authors Rahul Gupta and Rohit Gupta and generally, it has been applied in different areas of science and engineering [5, 6]. The response of electrical networks is generally obtained by the different mathematical approaches like the calculus approach [1-3], convolution theorem approach [7], or by various integral transforms like Laplace Transform [1-3], Mohand Transform [8, 9], Aboodh Transform [10], Elzaki Transform [11], residue theorem approach [12], Rohit Transform [13]. This paper presents the use of a new integral transform called Gupta Transform for obtaining the complete response of the series RL and RC networks circuits with a steady voltage source and brings up the Gupta Transform as a new successful powerful tool for determining the response of network circuits.

2. Gupta Transform

Let $g(y)$ is a continuous function on any interval for $y \geq 0$. The Gupta Transform of $g(y)$ is as [5, 6]

$\mathring{R}\{g(y)\} = \frac{1}{q^3} \int_0^\infty e^{-qy} g(y) dy = G(q)$, provided that the integral is convergent, where, q may be a real or complex parameter and \mathring{R} is the Gupta Transform operator. The Gupta Transform of elementary functions are given in [5, 6, 15]. The inverse Gupta Transform of the function $G(r)$ is denoted by $\mathring{R}^{-1}\{G(r)\}$ or $g(y)$. If we write $\mathring{R}\{g(y)\} = G(r)$, then $\mathring{R}^{-1}\{G(r)\} = g(y)$, where \mathring{R}^{-1} is called the inverse Gupta Transform operator. The Inverse Gupta Transform of elementary functions are given in [5, 6]. The Gupta Transform of some derivatives [5, 6, 15] of $g(y)$ are,

$$\mathring{R}\{g'(y)\} = q\mathring{R}\{g(y)\} - \frac{1}{q^3}g(0),$$

$$\mathring{R}\{g''(y)\} = q^2\mathring{R}\{g(y)\} - \frac{1}{q^2}g(0) - \frac{1}{q^3}g'(0).$$

2.1 Series RL Network Circuit with Steady Voltage Source

We will take a series RL network circuit to which a steady voltage source of potential V_0 is applied through a key K as shown in figure 1.

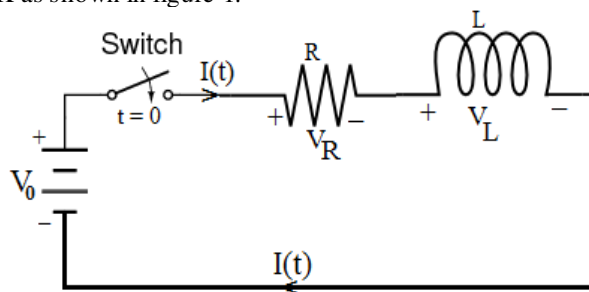


Fig.1: Series RL network with steady voltage source

As the switch is closed at $t = 0$, the potential drops across the network elements are given by [1-4].

$$V_R(t) = I(t)R, \quad V_L(t) = L I'(t).$$

Therefore, the application of Kirchoff's loop law to the loop shown in figure 2 provides

$$V_R(t) + V_L(t) = V$$

Or

$$R I(t) + L [I'(t)] = V_0 \quad (1)$$

Differentiating (1), a differential equation of order 2 as given below:

$$I''(t) + \frac{R}{L} I'(t) = 0 \quad (2)$$

To solve equation (2), we first write the initial conditions as follows [1-4]:

- Since the current in the inductor and the electric potential difference across the capacitor cannot change instantaneously, therefore, as the switch is closed at the instant $t = 0$, then $I(0) = 0$.
- Since $I(0) = 0$, therefore, (1) provides $[I'(0)] = \frac{V_0}{L}$.

The Gupta Transform [5, 6, 15] of (2) provides

$$q^2 \bar{I}(q) - \frac{1}{q^2} I(0) - \frac{1}{q^3} I'(0) + \frac{R}{L} \left\{ q \bar{I}(q) - \frac{1}{q^3} I(0) \right\} = 0 \quad (3)$$

Applying $I(0) = 0$ and $[I'(0)] = \frac{V_0}{L}$, (3) becomes,

$$q^2 \bar{I}(q) - \frac{1}{q^3} \frac{V_0}{L} + \frac{R}{L} q \bar{I}(q) = 0$$

Or

$$\bar{I}(q) = \frac{V_0}{L} \left[\frac{1}{q^3} \frac{1}{q(q + \frac{R}{L})} \right] \quad (4)$$

This equation can be rewritten as

$$\bar{I}(q) = \frac{V_0}{R} \left[\frac{1}{q^4} - \frac{1}{q^3} \frac{1}{q + \frac{R}{L}} \right]$$

Taking inverse Gupta Transform [5, 6], we have

$$I(t) = \frac{V_0}{R} \left[1 - e^{-\left(\frac{R}{L}\right)t} \right] \quad (5)$$

This equation (5) provides the complete response of the series RL network circuit with a steady potential source.

2.2 Series RC Network Circuit with Steady Voltage Source

The series R-C network circuit with a steady potential source (as show in figure 2) is analyzed by the following equation [1-3]

$$\dot{Q}(t)R + \frac{Q(t)}{C} = V_0$$

Or

$$\dot{Q}(t) + \frac{1}{RC} Q(t) = \frac{V_0}{R} \quad (6)$$



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Here, $Q(t)$ is the instantaneous charge and $Q(0) = 0$.
The Gupta Transform [5, 6, 15] of (6) gives

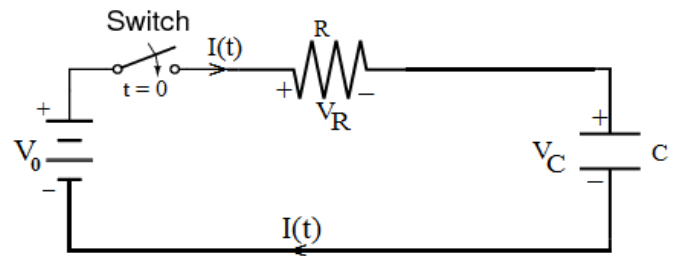


Fig.2: Series RC network with steady voltage source

$$\left\{ q \bar{Q}(q) - \frac{1}{q^3} Q(0) \right\} + \frac{1}{RC} \bar{Q}(q) = \frac{V_0}{R}$$

Put $Q(0) = 0$ we get

$$\left\{ q \bar{Q}(q) \right\} + \frac{1}{RC} \bar{Q}(q) = \frac{V_0}{R} \frac{1}{q^3}$$

This equation can be rewritten as

$$\bar{Q}(q) = \frac{V_0}{R} \frac{1}{q^3 \left(q + \frac{1}{RC} \right)}$$

Or

$$\bar{Q}(q) = CV_0 \left[\frac{1}{q^4} - \frac{1}{q^3} \frac{1}{q + \frac{1}{RC}} \right] \quad (7)$$

Taking inverse Gupta Transform [5, 6], we have

$$Q(t) = CV_0 \left[1 - e^{-\left(\frac{1}{RC}\right)t} \right] \quad (8)$$

This equation (8) provides the complete response of the series RC network circuit with a steady potential source.

3. Conclusion

In this paper, a new integral transform called Gupta Transform has been successfully applied for determining the complete response of the series RL and RC networks with a steady excitation source. The results obtained are the same as obtained with other approaches [1-3, 14]. This paper brought up the Gupta Transform as a new successful powerful tool for determining the response of network circuits.

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