Engineering and Scientific International Journal (ESIJ)
Volume 2, Issue 3, July – September 2015
ISSN 2394 - 7179 (Print)
ISSN 2394 - 7187 (Online)

Quasi-stationary thermoelastic problem with moving heat source in unidirectional Dirichlet's rod

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Abstract— In this paper we deal quasi-stationary, non-homogeneous thermo elastic problem with Dirichlet's boundary condition in one dimensional rod of isotropic material occupying the region $0 \le x \le a$. Initial temperature of the rod is zero and placed in an ambient temperature zero. The rod is subjected to the activity of moving point heat source located at x' moving with constant velocity u along x-axis.

Keywords**—**Thermoelastic problem, Quasi-stationary, Moving heat source, Dirichlet's boundary condition

1. Introduction

Newly we have studied transient non- homogeneous thermoelastic problem in skinny rectangular plate with Dirichlet's, Robin's and Neumann's boundary condition from [1] to [10]. Now in this paper authors decide the temperature distribution and thermal stresses in one dimension with moving point heat source with quasi-stationary condition. This is new contribution in the field of thermoelasticity.

2. Formulation of the Problem

Consider a one dimensional rod of length a occupying the region $0 \le x \le a$. Initial temperature of the rod is zero placed in an ambient temperature zero. The rod is subjected to the activity of instantaneous moving point heat source at the point x' which changes its place along x axis, moving with constant velocity u . The activity of moving heat source in the rod cause the generation of heat due to nuclear interaction that be a function of position and time in the form $g(x,t)\, w/s^3$. The temperature sharing of the rod in one dimension is described by the differential equation of heat conduction with heat generation term, as in [11] page 8, is given by

$$\frac{\partial^2 T}{\partial x^2} + \frac{1}{k} g = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

Where $T = T(x,t)$ is temperature distribution, k is the thermal conductivity of the material of the rod,

$$\alpha = \frac{k}{\rho c_p}$$

is thermal diffusivity, ρ is density, c_p is specific heat of the material of the rod and g is energy (heat) generation term. Now consider an instantaneous moving point heat source located at point x' and releasing its heat spontaneously at time t. Such point moving heat source in one dimension rod is given by delta function

$$g(x,t) = g_i^{\,p}\,\delta(x - x')$$

Where,

$$x' = ut$$

Hence above equation reduces to

$$\frac{\partial^2 T}{\partial x^2} + \frac{1}{k} g_i^{\,p}\,\delta(x - ut) = \frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{2.1}$$

With initial and boundary condition

$$T = 0 \text{ at } x = 0, t = 0 \tag{2.2}$$

$$T = 0 \text{ at } x = a, t = 0 \tag{2.3}$$

In the solution of the moving heat source problem it is convenient to let the coordinate system move with the source. This is achieved by introducing a new coordinate X defined by

$$X = x - ut \tag{2.4}$$

Hence above equation reduces to

$$\frac{\partial^2 T}{\partial X^2} + \frac{1}{k} g_p^{\,i}\,\delta(X) = \frac{1}{\alpha}\left[\frac{\partial T}{\partial t} - u\frac{\partial T}{\partial X}\right] \tag{2.5}$$

3. Quasi-Stationary Condition

Stationary medium means initially at zero temperature. Hence quasi-stationary condition is mathematically defined by setting $\frac{\partial T}{\partial t} = 0$. Hence (2.5) reduces to

$$\frac{\partial^2 T}{\partial X^2} + \frac{u}{\alpha}\frac{\partial T}{\partial X} = -\frac{1}{k} g_p^{\,i}\,\delta(X) \tag{3.1}$$

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4. Solution

The solution of (3.1) is

$$T(x,t) = A + Be^{-\frac{u}{\alpha}x} e^{\left(\frac{u^2}{\alpha}t\right)} + \frac{\alpha}{ku} g_p^i \left[e^{-\frac{u}{\alpha}x} e^{\frac{2u^2}{\alpha}t} - 1 \right]$$

Applying the condition (2.2) and (2.3) we obtain

$$A = \frac{\alpha}{ku} g_p^i \qquad B = -\frac{\alpha}{ku} g_p^i \tag{4.2}$$

From (4.1) and (4.2) we obtain

$$T(x,t) = \frac{\alpha}{ku} g_p^i e^{-\frac{u}{\alpha}x} \left[e^{\frac{2u^2}{\alpha}t} - e^{\frac{u^2}{\alpha}t} \right]$$

5. Thermoelastic Problem

Let us introduce a thermal stress function χ related to component of stress in the material as in [12]

$$\sigma_{yy} = \frac{\partial^2 \chi}{\partial x^2} \tag{5.1}$$

The boundary condition

$$\sigma_{yy} = 0, \text{ at } x = 0, x = a \tag{5.2}$$

Where,

$\chi = \chi_c + \chi_p$, χ_c is complementary function and χ_p is particular function. χ_c and χ_p are governed by a linear homogeneous differential equation and linear non-homogeneous differential equation respectively and

$$\nabla^4 \chi_c = 0 \tag{5.3}$$

$$\nabla^2 \chi_p = -\lambda E \Gamma \tag{5.4}$$

Where,

$$\nabla^2 = \frac{\partial^2}{\partial x^2}$$

Where,

Γ is temperature change $\Gamma = T - T_i$ where T_i is initial temperature

6. Solution of Thermoelastic Problem

$$\Gamma = T - T_i$$

$$\Gamma = \frac{\alpha}{ku} g_p^i e^{-\frac{u}{\alpha}x} \left[e^{\frac{2u^2}{\alpha}t} - e^{\frac{u^2}{\alpha}t} \right] \tag{6.1}$$

Let the suitable form of χ_c satisfying (5.3) be

$$\chi_c = cx^3 + dx^2 \tag{6.2}$$

Let suitable form of χ_p satisfying (5.4) be

$$\chi_p = -\frac{\lambda E \alpha^3}{ku^3} g_p^i e^{\left(-\frac{u}{\alpha}x\right)} \left\{ e^{\left(\frac{2u^2}{\alpha}t\right)} - e^{\left(\frac{u^2}{\alpha}t\right)} \right\} \tag{6.3}$$

$$\chi = \chi_c + \chi_p = cx^3 + dx^2 - \frac{\lambda E \alpha^3}{ku^3} g_p^i e^{\left(-\frac{u}{\alpha}x\right)} \left\{ e^{\left(\frac{2u^2}{\alpha}t\right)} - e^{\left(\frac{u^2}{\alpha}t\right)} \right\} \tag{6.4}$$

From (5.1) and (6.4) we obtain

$$\sigma_{yy} = 6cx + 2d - \frac{\lambda E \alpha}{ku} g_p^i e^{\left(-\frac{u}{\alpha}x\right)} \left\{ e^{\left(\frac{2u^2}{\alpha}t\right)} - e^{\left(\frac{u^2}{\alpha}t\right)} \right\} \tag{6.5}$$

Applying (5.2) to (6.5) we obtain

$$d = \frac{1}{2} \frac{\lambda E \alpha}{ku} g_p^i \left\{ e^{\left(\frac{2u^2}{\alpha}t\right)} - e^{\left(\frac{u^2}{\alpha}t\right)} \right\} \tag{6.6}$$

$$c = \frac{\lambda E \alpha}{6aku} g_p^i \left\{ e^{\left(\frac{2u^2}{\alpha}t\right)} - e^{\left(\frac{u^2}{\alpha}t\right)} \right\} \left[e^{\left(-\frac{u}{\alpha}a\right)} - 1 \right] \tag{6.7}$$

From (6.5), (6.6) and (6.7) we obtain

$$\sigma_{yy} = \frac{\lambda E \alpha}{ku} g_p^i \left\{ e^{\left(\frac{2u^2}{\alpha}t\right)} - e^{\left(\frac{u^2}{\alpha}t\right)} \right\} \left\{ \frac{x}{a} \left[e^{\left(-\frac{u}{\alpha}a\right)} - 1 \right] - \left[e^{\left(-\frac{u}{\alpha}x\right)} - 1 \right] \right\} \tag{6.8}$$

7. Conclusion

In this paper we determined the time dependent temperature distribution and thermal stresses in one dimension, with moving heat source in stationary condition with analytical approach. By giving particular values to the parameters one can obtain their desired results by putting values of the parameters in the equations (4.3) and (6.8). From these equations we observe that initially ($t = 0$) all stresses vanish.

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