

# Heat and Mass Transfer of Steady Hydromagnetic Flow on a Continuously Moving Surface with Soret Effect and Thermophoretic Effect

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**Abstract**— In the present research paper Thermophoretic and Soret effects in steady free convection flow with magnetic field along with moving surface is studied. Thermophoresis effects is not negligible due to its important applications in engineering mainly for removing small particles from gas streams, in determining exhaust gas particle trajectories from combustion devices and in studying the material deposition on turbine blades. The non-linear boundary layer equations are converted into ordinary differential equation and applying Runge-Kutta shooting method. The effects of non-dimensional temperature parameter, Prandtl number, temperature exponent, magnetic parameter, Eckert number, Lewis number, Soret number and thermophoretic parameter have been studied. The velocity profile, temperature distribution and concentration distribution graphs are plotted and discuss in details.

**Keywords** — Heat Transfer; Mass Transfer; Hydromagnetic Flow; Soret Number; Thermophoretic Parameter.

## 1. Introduction

Hydromagnetic free convection flow over a moving surface has important applications in many engineering processes such as stellar and planetary magnetospheres, aeronautics etc. The study of convective flow with mass transfer along a vertical porous plate is receiving considerable attention of the many researchers due to its varied applications within the field of comical and geophysical sciences. Thermophoresis helps in removing micro particles from gas stream used as thermal precipitators. It also helps in production of fine cermic powders. Many workers have considered the problem of free convection flow with mass transfer. Muthucumaraswamy and Kumar [1] have considered the thermal radiation effects on moving infinite vertical plate in the presence variable temperature and mass diffusion. Kandasamy [2] has considered the steady flow of an incompressible, viscous, electrically conducting and Boussinesq fluid over an accelerating vertical plate with heat source and thermal stratification effect in the presence of a uniform transverse magnetic field. The effect of temperature dependent viscosity on laminar mixed convection boundary layer flow and heat transfer on a continuously moving vertical surface has been studied by Ali [3]. Muthucumaraswamy et al. [4] have studied the radiative heat and mass transfer effects on moving isothermal vertical plate in the presence of chemical reaction. Alam et al. [5] has investigated the viscous dissipation effects on MHD natural convection flow over a sphere in the presence of heat generation. Mahmoud [6] has investigated the variable viscosity effects on hydromagnetic boundary layer flow along a continuously moving vertical plate in the presence of radiation. The flow due to a moving extensible sheet that obeys a more general

stretching law has been considered by Ishak et al. [7]. Double diffusive mixed convection steady laminar flow over a moving semi infinite vertical plate in the presence of internal heat generation and a chemical reaction has investigated by Patil et al. [8]. Double-Diffusive convection-radiation interaction on shaky MHD run over a vertical moving porous plate with heat cohort and Soret effects in the occurrence of porous medium has been deliberated by Mohamed [9].

Thermal radiation effects on free convection flow moving infinite vertical plate with uniform heat flux is studied by Chandrakala [10]. Heat transfer effects were taken into account and the dimensionless governing equations were solved using the Laplace-transform technique. Jat and Chaudhary [11] have studied the flow and heat transfer for an electrically conducting fluid past a continuously moving plate with variable surface temperature in the presence of a uniform transverse magnetic field. The effects of variable viscosity on hydro magnetic boundary layer flow along a continuously moving vertical plate in the presence of radiation and chemical reaction with uniform suction and heat flux has been studied by Das [12]. The radiation effects on free convection flow bounded by an impulsively started infinite vertical plate embedded in porous medium is investigated by Pathak and Sisodia [13]. The thermal-diffusion and diffusion-thermo effects on the warmth and mass transfer characteristics of free convection past a moving vertical plate embedded during a porous medium within the presence of magnetic flux, blowing and thermal radiation is discussed by Olanrewaju and Adeniyani [14]. The combined effects of Soret and Dufour on unsteady hydromagnetic free convective flow of a Newtonian, viscous, electrically conducting fluid on a continuously fluid past vertical absorbent plate suction in presence of

radiation absorption, chemical reaction and heat source parameter have been studied by Babu et al. [15]. Viscous incompressible fluid's two dimensional hydromagnetic forced convection boundary layer flows is studied by Uddin and Ali [16].

To study the consequences on both momentum and warmth transfer problem with viscous dissipation and Joule heat transfer for an electrically conducting fluid past a continuously moving plate within the presence of a consistent transverse magnetic flux is studied by Dakshinamoorthy et al. [17]. But the literature in which the combined effects of heat and mass transfer along with Soret-Dufor and Thermophoresis parameters are very few. Therefore it is proposed the combined effect of heat and mass transfer in steady flow of an electrically conducting, viscous incompressible fluid past a continuously moving surface with Soret effect and thermophoretic effect is presented in this paper.

## 2. Mathematical Formulation

Consider the two-dimensional steady flow of an electrically conducting, viscous, incompressible fluid past a continuously moving surface with uniform velocity  $U$  in the presence of uniform transverse magnetic field of strength  $B_0$  with Soret effect and thermophoretic effect.

The  $x$ - axis is taken along the surface and  $y$ -axis normal to it. The fluid properties are assumed to be isotropic and constant, except for the fluid viscosity which is assumed to be an inverse linear function of temperature, Lai et al. [18], as given by:

$$\bar{\mu} = \frac{\mu_\infty}{(1 + \gamma(T - T_\infty))} \quad \text{or} \quad \frac{1}{\bar{\mu}} = \alpha(T - T_\Gamma),$$

with  $\alpha = \frac{\gamma}{\mu_\infty}$ ;  $T_\Gamma = T_\infty - \frac{1}{\gamma}$ , (1)

In the above equation (1),  $T$  and  $T_\infty$  are the temperature of fluid near and far away from the moving plate.  $\alpha$  and  $T_\Gamma$  are constants and their values depend on the reference state and thermal property of fluid. In general,  $\alpha > 0$  for liquids and  $\alpha < 0$  for gases.  $\mu$  is known as coefficient of viscosity and  $\mu_\infty$  is known as reference viscosity,  $\gamma$  is constant. The boundary layer equations governing the flow and heat transfer due to a continuously moving surface are:

Equation of continuity,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (2)$$

Equation of momentum,

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{1}{\rho} \frac{\partial}{\partial y} \left( \mu \frac{\partial u}{\partial y} \right) - \frac{\sigma_e B_0^2 u}{\rho} \quad (3)$$

Equation of heat transfer,

$$\rho C_p \left( u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \frac{\partial^2 T}{\partial y^2} + \mu \left( \frac{\partial u}{\partial y} \right)^2 + \sigma_e B_0^2 u^2 \quad (4)$$

Equation of mass transfer,

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} + \frac{Dk_T}{T_m} \frac{\partial^2 T}{\partial y^2} - \frac{\partial}{\partial y} [V_T (C - C_\infty)] \quad (5)$$

and corresponding boundary conditions are :

$$y = 0: \quad u = U, \quad v = 0, \quad T = T_w(x), \quad C = C_w(x)$$

$$y \rightarrow \infty: \quad u = 0, \quad T \rightarrow T_\infty, \quad C \rightarrow C_\infty. \quad (6)$$

Where,  $x$  and  $y$  represent the coordinate axes along the moving surface in longitudinal and normal to it,  $u$  and  $v$  are the components of velocity along the  $x$  and  $y$  axes,  $\sigma_e$  is the electrical conductivity,  $B_0$  is the applied magnetic strength,  $C$  is used to represent fluid concentration,  $\rho$  represents fluid density,  $C_\infty$  shows fluid free stream concentration,  $k$  shows variable thermal conductivity,  $k_T$  is the thermal diffusion ratio,  $T_m$  is the mean fluid temperature,  $B_0$  is the magnetic field intensity,  $v_T$  is the thermophoretic velocity and  $D$  is the molecular diffusivity of the species concentration.

## 3. Method of Solution

We introduce a dimensional stream function  $\psi(x, y)$ , defined as

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \quad (1)$$

which satisfies the continuity equation (2). The dimensionless variables are introduced to obtain the similarity solutions for the equations (3) to (5) with boundary conditions (6) are,

$$\psi = \sqrt{U x v_\infty} f(\eta), \quad \eta = y \sqrt{\frac{U}{v_\infty x}}, \quad T_w(x) - T_\infty = Ax^n$$

$$\theta = \frac{(T - T_\infty)}{(T_w - T_\infty)}, \quad \phi = \frac{(C - C_\infty)}{(C_w - C_\infty)}, \quad \theta_\Gamma = \frac{(T_\Gamma - T_\infty)}{(T_w - T_\infty)} \quad (7)$$

In boundary layer theory, it is said that the temperature gradient is very lower along the surface as compare to temperature gradient normal to the surface, i.e.  $\frac{\partial T}{\partial y} \gg \frac{\partial T}{\partial x}$ , (2)

So the component of thermophoretic velocity along the plate is negligible. As a result, the thermophoretic

velocity  $V_T$ , which appears in equation (5), can be written as:

$$V_T = -\frac{\beta v}{T_{ref}} \frac{\partial T}{\partial y} \quad (8)$$

where  $\beta$  is the thermophoretic coefficient which ranges in value from 0.2 to 1.2 as indicated by Batchelor and Shen [19] and is defined from the theory of Talbot et al. [20] by:

$$\beta = \frac{2C_s(\lambda_g/\lambda_p + C_t Kn)[1 + Kn(C_1 + C_2 e^{-C_3/Kn})]}{(1 + 3C_m Kn)(1 + 2\lambda_g/\lambda_p + 2C_t Kn)} \quad (9)$$

where  $C_1, C_2, C_3, C_s, C_m$  and  $C_t$  are constants,  $\lambda_g$  and  $\lambda_p$  are the thermal conductivities of the fluid and diffused particles, respectively and  $Kn$  is the Knudsen number.

A thermophoretic parameter  $\tau$  can be defined (see Mills et al. [21] and Tsai [22]) as follows:

$$\tau = -\frac{\beta(T_w - T_\infty)}{T_{ref}} \quad (10)$$

Typical values of  $\tau$  are 0.01, 0.1 and 1.0 corresponding to approximate values of  $-\beta(T_w - T_\infty)$  equal to 3, 30 and 300 K for a reference temperature of  $T_{ref} = 300$  K.

The equation of continuity is identically satisfied with the above transformations and momentum, heat and mass transfer equations (3)-(5) is reduced to

$$f'' - \left(\frac{\theta'}{\theta - \theta_r}\right) f' - \left(\frac{\theta - \theta_r}{2\theta_r}\right) f f'' + \left(\frac{\theta - \theta_r}{\theta_r}\right) Re_m^2 f' = 0 \quad (11)$$

$$\theta'' - n Pr f \theta + \frac{Pr}{2} \theta' f - Pr \left(\frac{\theta_r}{\theta - \theta_r}\right) Ec f'^2 + Re_m^2 Pr Ec f'^2 = 0 \quad (12)$$

$$\frac{1}{Le} \varphi'' + Sr \theta' + \frac{Pr}{2} f \varphi' - \tau (\varphi' \theta + \varphi \theta'') = 0 \quad (13)$$

With the boundary conditions

$$\begin{aligned} \eta = 0: & \quad f = 0, f' = 1, \theta = 1, \varphi = 1 \\ \eta \rightarrow \infty: & \quad f' \rightarrow 0, \theta \rightarrow 0, \varphi \rightarrow 0, \end{aligned} \quad (14)$$

where prime (') denotes the derivative with respect to  $\eta$ ,  $A$  denotes the stratification rate of the gradient of ambient temperature,  $n$  is the temperature exponent,

$$Sr = \frac{Dk_T(T_w - T_\infty)}{T_m \alpha (C_w - C_\infty)} \text{ is the Soret number, } Pr = \frac{\mu_\infty C_p}{k}$$

is the Prandtl number,  $Le = \frac{\alpha}{D}$  is the Lewis number,

$$Re_m = B_0 \sqrt{\frac{\sigma_e x}{\rho U}} \text{ is the Reynolds magnetic parameter and}$$

$$Ec = \frac{U^2}{C_p (T_w - T_\infty)} \text{ is the Eckert number.}$$

#### 4. Numerical Computation

The transformed non-linear coupled ordinary differential equations (11) to (13) with boundary conditions (14) are solved with the help of Runge-Kutta method with shooting technique.

First, the boundary value problem is converted to an initial value problem as, let

$$f(\eta) = y_1, \quad f'(\eta) = y_2, \quad f''(\eta) = y_3$$

$$\theta(\eta) = y_4, \quad \theta'(\eta) = y_5$$

$$\varphi(\eta) = y_6, \quad \varphi'(\eta) = y_7$$

Then equations (11) to (13), reduce into a system of ordinary differential equations of first order, viz,

$$y_1' = y_2, \quad y_1(0) = 0$$

$$y_2' = y_3, \quad y_2(0) = 1$$

$$y_3' = \left(\frac{y_5}{y_4 - \theta_r}\right) y_3 + \left(\frac{y_4 - \theta_r}{2\theta_r}\right) y_1 y_3 - \left(\frac{y_4 - \theta_r}{\theta_r}\right) Re_m^2 y_2,$$

$$y_3(0) = m^{(1)}$$

$$y_4' = y_5, \quad y_4(0) = 1$$

$$y_5' = n Pr y_2 y_4 - \frac{Pr}{2} y_5 y_1 + Pr \left(\frac{\theta_r}{y_4 - \theta_r}\right) Ec y_3^2 - Re_m^2 Pr Ec y_2^2,$$

$$y_5(0) = m^{(2)}$$

$$y_6' = y_7, \quad y_6(0) = 1$$

$$y_7' = -Le[(Sr - \tau y_6) \{ nPr y_2 y_4 - \frac{Pr}{2} y_5 y_1 + Pr(\frac{\theta_r}{y_4 - \theta_r}) Ec y_3^2 - Re_m^2 Pr Ec y_2^2 \} + (\frac{Pr}{2} y_1 - \tau y_5) y_7],$$

$$y_7(0) = m^{(3)} \tag{15}$$

Here  $m^{(1)}$ ,  $m^{(2)}$  and  $m^{(3)}$  have taken guess in such a way that it satisfy  $y_2(\infty) = 0, y_4(\infty) = 0$  and  $y_6(\infty) = 0$ . In this method, the boundary value problem (11) to (13) is reduced to an initial value problem (15). The initial values  $m^{(1)}$ ,  $m^{(2)}$  and  $m^{(3)}$  are guessed until the boundary value conditions  $y_2(\infty) = 0, y_4(\infty) = 0$  and  $y_6(\infty) = 0$  are satisfied. Finally the resulting equations are solved with the help of Runge-Kutta integration scheme for  $\Delta\eta = 0.01$  and at  $\eta(\infty) = 7$ , by initial value solver of computational software using MATLAB.

### 5. Results and Discussion

The influence of the various parameters ( $\theta_r, Pr, n, Ec, Le, Sr, \tau$  and  $Re_m$ ) involved in the problem are graphically presented on the velocity, temperature and concentration profiles. In plotting the results for  $\eta \rightarrow \infty$  as  $\eta_{max} = 7$  and step size  $\Delta\eta = 0.01$  were used in the boundary conditions. It is important to note that  $\theta_r$  is negative for liquids ( $Pr > 1.0$ ) and positive for gases ( $Pr < 1.0$ ). In order to get insight the problem, the following values of the parameters are considered  $Re_m = 0.09$ ,  $Ec = 0.5$ ,  $Sr = 1$ ,  $\theta_r = 2$ ,  $Pr = 0.02$ ,  $Le = 0.2$ ,  $\tau = 0.1$  and temperature exponent  $n = 0.3$ .

The effects due to the temperature exponent ( $n$ ) on the velocity and temperature distributions are shown in figures 1 and 2, respectively. It was observed that the velocity and temperature decrease as the temperature exponent increases, while there is no significant effect of temperature exponent on concentration distribution.

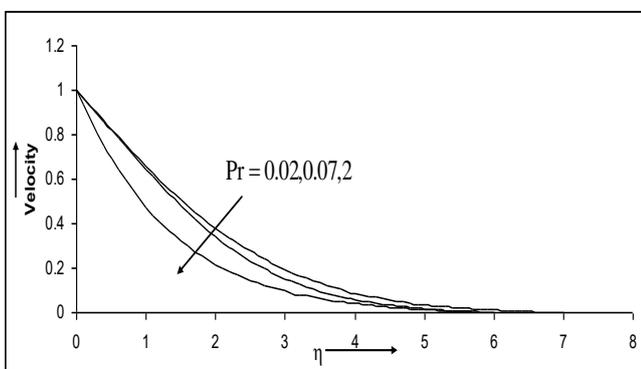


Fig. 1: Velocity profiles for various values of temperature exponent

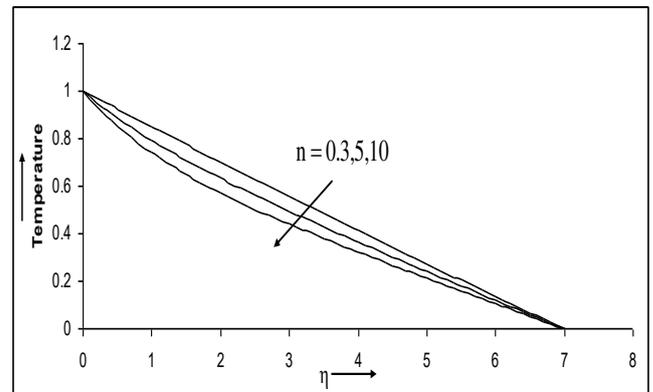


Fig. 2: Temperature profiles for various values of temperature exponent (n)

Figures 3, 4 and 5 depict the velocity, temperature and concentration profiles for different values of the Prandtl number ( $Pr$ ). It is clear that the velocity, temperature and concentration decrease with an increase in the Prandtl number from .02 to 2.0.

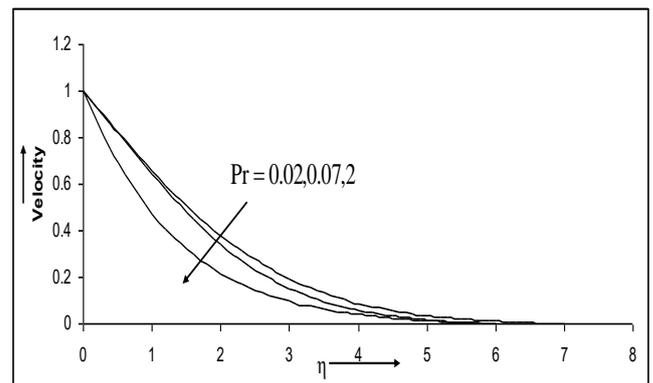


Fig. 3: Velocity profile for various values of Prandtl number

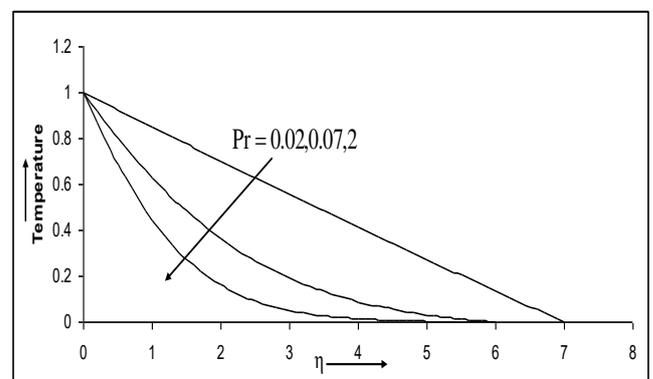


Fig. 4: Temperature profile for various values of Prandtl number

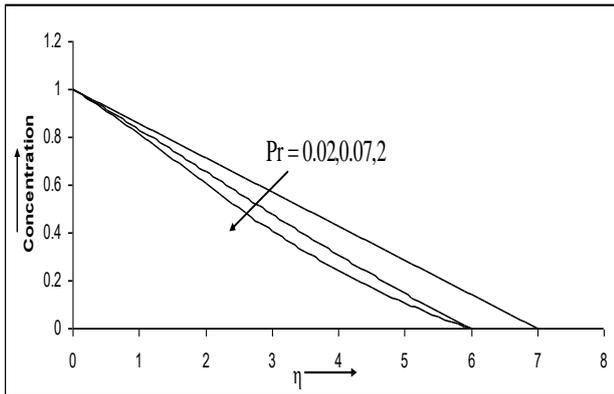


Fig. 5: Concentration profile for various values of Prandtl number

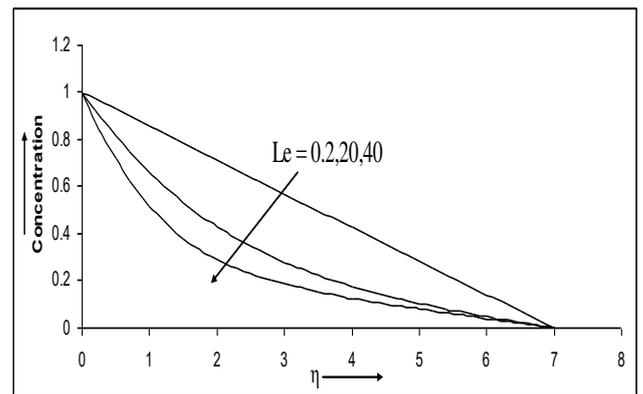


Fig. 8: Concentration profile for various values of Lewis number

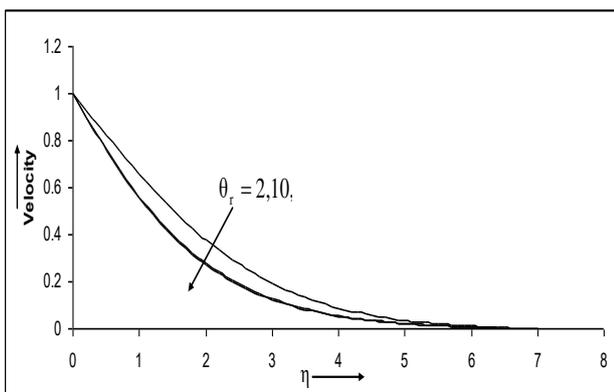


Fig. 6: Velocity profile for various values of temperature parameter

Figure 6 depict the dimensionless velocity profiles for different values of the dimensionless temperature. It is observed that the velocity of the fluid decreases with increase of the dimensionless temperature while the temperature and the concentration of the fluid remain unchanged with increase of the dimensionless temperature.

In Figure 7, the dimensionless velocity profiles for different values of the Reynolds magnetic parameter are plotted. It is observed that the velocity of the fluid decreases with increase of the Reynolds magnetic parameter. In the presence of Lewis number, it is seen that the increase in the Lewis number, leads to a fall in the concentration of the fluid along the accelerating surface and is shown in Figure 8.

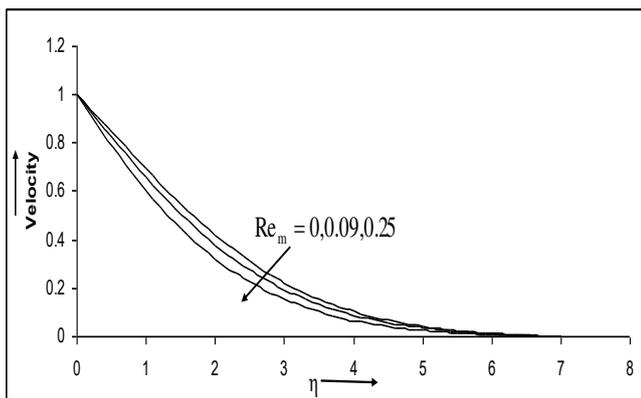


Fig. 7: Velocity profile for various values of Reynolds magnetic parameter

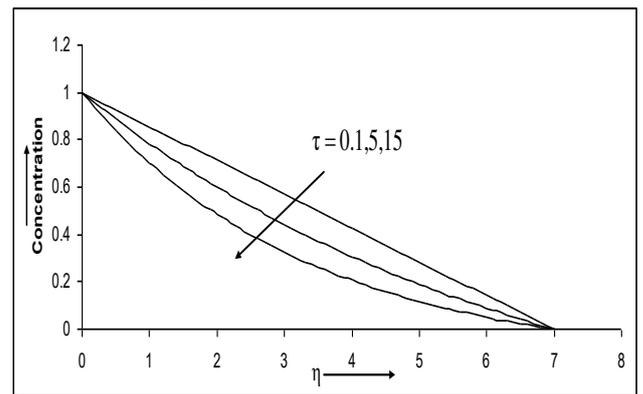


Fig. 9: Concentration profile with thermophoretic parameter

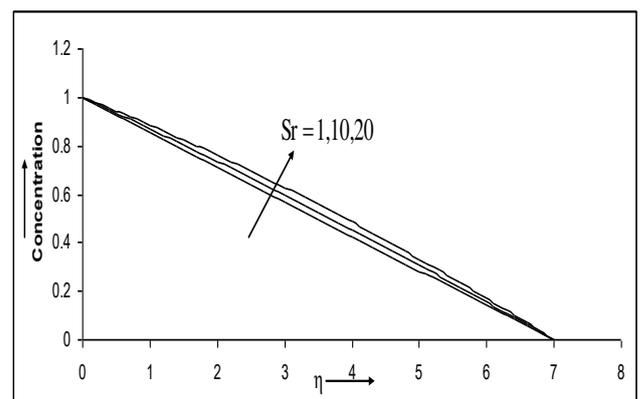


Fig. 10: Concentration profile with Soret number

In figures 9 and 10, the concentration profiles are plotted with different values of thermophoretic parameter and Soret number. It is observed that the increase in the thermophoretic parameter, leads to a fall in the concentration of the fluid and increase in the Soret number a rise in the concentration of the fluid along the accelerating surface and are shown through figures 9 and 10, respectively. There is no significant effect of Lewis number, Soret number and thermophoretic parameter on velocity and temperature distribution.

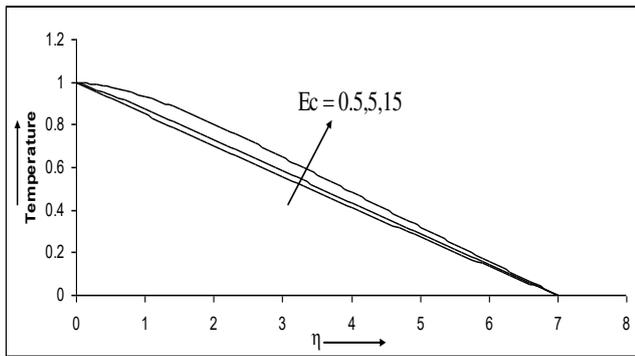


Fig. 11: Temperature profile with Eckert number

Figure 11 depicts the temperature profile for different values of the Eckert number with other parameters. It is observed that the temperature of the fluid increases as Eckert number increases.

## 6. Conclusions

In this paper we investigated the thermophoretic and Soret effects in steady free convection flow along with moving surface. From the present study we can make the following conclusions:

- Velocity profile decreases with the increasing values of temperature exponent  $n$  and temperature parameter  $\theta_r$ .
- Temperature profile decreases with the increasing values of Prandtl number  $Pr$  and temperature exponent  $n$ . Temperature boundary layer increases with the increasing values of the Eckert number.
- Velocity profiles decreases with increasing values Reynolds magnetic parameter  $Re_m$ .
- Concentration within the boundary-layer decreases with the increasing values of the thermophoretic parameter and Lewis parameter.
- Concentration profiles increases with the increasing values of the Soret number.

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